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Inpainting of Binary Images Using the Cahn-Hilliard Equation

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Abstract—**Image inpainting is the filling in of missing or damaged regions of images using information from surrounding areas. The use of a model for inpainting based on the Cahn-Hilliard equation, which allows for fast, efficient inpainting of degraded text, as well as super-resolution of high contrast images is introduced. Image decomposition is an important problem in image processing. The use of a model for inpainting based on the Cahn-Hilliard equation, which allows for fast, efficient inpainting of degraded text, as well as super-resolution of high contrast images is introduced. Cahn-Hilliard equation gives a better result on gray scale images. The Cahn-Hilliard equation is a nonlinear fourth order diffusion equation originating in material science for modeling phase separation and phase coarsening in binary alloys. The inpainting of binary images using the Cahn-Hilliard equation is a new approach in image processing. Processing is done by using subgradients of the total variation functional within the flow, which leads to structure inpainting with smooth curvature of level**

Index Terms—**Classification; descriptors; Foreground support; Histogram of oriented gradients; low rank modeling; Moving object detection;**

I. INTRODUCTION

Image decomposition is an important problem in image processing. It plays a significant role in realm of object recognition, biomedical engineering, astronomical imaging, etc. The target image is required to be decomposed into two meaningful components. One is the geometrical part or sketchy approximation of an image which is called cartoon component, and the other is the oscillating part or small scale special patterns of an image which is called texture component.

Mathematically, the cartoon component can be described by a piecewise smooth (or a piecewise constant) function whilst the texture component is commonly oscillating. Because of their different properties, it is more efficient and effective to separate them for image processing and image analysis. The main task herein is to extract the cartoon and texture components from a given image with degradation, e.g., blurry and/or missing pixels. For a target image $f \in R$, the image decomposition is to derive $f = u+v$, where u and v represent the cartoon and texture, respectively. A two-dimensional or higher dimensional image can be vectorizied as a one-dimensional vector e.g., in lexicographic order.

A decomposition model to restore blurred images with missing pixels is developed. And here the assumption is that the underlying image is the superposition of cartoon and texture components. The total variation norm and its dual norm is used to regularize the cartoon and texture, respectively. Numerical algorithm based on the splitting versions of augmented Lagrangian method is used to solve the problem. The existence of a minimizer to the energy function and the convergence of the algorithm are guaranteed. In contrast to recently developed methods for deblurring images,

the proposed algorithm not only gives the restored image, but also gives a decomposition of cartoon and texture parts. These two parts can be further used in segmentation and inpainting problems.

Image inpainting is the filling in of damaged or missing regions of an image with the use of information from surrounding areas. In its essence, it is a type of interpolation. Its applications include restoration of old paintings by museum artists, and removing scratches from photographs. For instance, in the museum world, in the case of a valuable painting, this task would be carried out by a skilled art conservator or art restorer. In the digital world refers to the application of sophisticated algorithms to replace lost or corrupted parts of the image data (mainly small regions or to remove small defects). In photography and cinema, is used for film restoration; to reverse the deterioration (e.g., cracks in photographs or scratches and dust spots in film; see infrared cleaning). It is also used for removing red-eye, the stamped date from photographs and removing objects to creative effect.

This technique can be used to replace the lost blocks in the coding and transmission of images, for example, in a streaming video. It can also be used to remove logos in videos. Inpainting is rooted in the restoration of images. Traditionally, inpainting has been done by professional restorers. The underlying methodology of their work is as follows:

The global picture determines how to fill in the gap. The purpose of inpainting is to restore the unity of the work. The structure of the gap surroundings is supposed to be continued into the gap. Contour lines that arrive at the gap boundary are prolonged into the gap. The different regions inside a gap, as defined by the contour lines, are filled with colors matching for those of its boundary.

CAHN-HILLIARD equation is used for inpainting the image. The Cahn–Hilliard equation (after John. W.Cahn and John E. Hilliard)is an equation of mathematical physics which describes the process of phase separation, by which the two components of a binary fluid spontaneously separate and form domains pure in each component

The rest of this paper is organized as follows. Section II describes the previous works. Section III describes the proposed methodology in detail. Finally, conclusion is given in Section VI.

II. LITERATURE SURVEY

The following section describes some of the previous works on image inpainting.

(L. Rudin, S. Osher, and E. Fatemi 1992) proposed first model for image decomposition. The presence of noise in images is unavoidable. It may be introduced by the image formation process, image recording, image transmission, etc. These random distortions make it difficult to perform any required picture processing. The feature oriented enhancement introduced is very effective in restoring blurry images, but it can be "frozen" by an oscillatory noise component. Even a small amount of noise is harmful when high accuracy is required, e.g. as in subcell (subpixel) image analysis. In practice, to estimate a true signal in noise, the most frequently used methods are based on the least squares criteria. The rationale comes from the statistical argument that the least squares estimation is the best over an entire ensemble of all possible pictures.

(Daubechies and G. Teschke 2002) proposed a wavelet-based treatment of variational problems arising in the field of image processing. A special class of variational functionals that induce a decomposition of images into oscillating and cartoon components and possibly an appropriate ‘noise’ component. the cartoon component of an image is modeled by a *BV* function; the corresponding incorporation of *BV* penalty terms in the variational functional leads to PDE schemes that are numerically intensive. By replacing the *BV* penalty term by a $L1$ term (which amounts to a slightly stronger constraint on the minimizer), and writing the problem in a wavelet framework, we obtain elegant and numerically efficient schemes with results very similar to those obtained in Modeling textures with total variation minimization and oscillating patterns in image processing. It mainly focuses on a special class of variational problems which induce decomposition of images in oscillating and cartoon components; the cartoon part is ideally piecewise smooth with possible abrupt edges and contours; the oscillation part, on the other hand, ‘fills’ in the smooth regions in the cartoon with texture-like features. Several authors, propose to model the cartoon component by the space *BV* which induces a penalty term that allows edges and contours in the reconstructed cartoon images. However, the minimization of variational problems of this type usually results in PDE-based schemes that are numerically intensive.

(L. Vese and S. Osher 2003) proposed a system that Performs modeling of real textured images by functional minimization and PDE. Combines ROF method with texture preserving model. To model texture component space of oscillating functions are used, finite differential method is used here to obtain the decomposition U+V.

(F. Aujol, G. Gilboa, T. Chan, and S. Osher,2006) This paper explores various aspects of the image decomposition problem using modern variational techniques. We aim at splitting an original image f into two components u and v, where u holds the geometrical information and v holds the textural information. The focus of this paper is to study energy terms and functional spaces that suit various types of textures. Our modeling uses the total-variation energy for extracting the structural part and one of four of the following norms for the textural part: L2, G, L1 and a new tunable norm, suggested here for the first time, based on Gabor functions. Apart from the broad perspective and our suggestions when each model should be used, the paper contains three specific novelties: first we show that the correlation graph between u and v may serve as an efficient tool to select the splitting parameter, second we propose a new fast algorithm to solve the TV -L1 minimization problem, and third we introduce the theory and design tools for the TV -Gabor model

(J.-F. Cai, R. H. Chan, and Z. Shen,2010) introduced the problem of inpainting, process is to fill-in the missing part in images. It is an interesting and important inverse problem. It arises, for example, in removing scratches in photos, in restoring ancient drawings, and in filling in the missing pixels of images transmitted through a noisy channel. We need to extract information such as edges and textures from the observed data to fill in the missing part such that shapes and patterns are consistent in the human vision. One popular approach for image inpainting is the PDE-based method. The idea is to propagate the geometric information of the curves along the edges by specially designed differential operators. Since the PDE-based approaches are able to keep the edges, it performs very well for piecewise smooth images. Mainly focusing on minimization problems in image inpainting that simultaneously restore the cartoon and texture parts of the image. By using proximal forward-backward splitting, we have proposed algorithms that solve the minimization problems, and established their convergence. Numerical examples are given to illustrate the applicability and usefulness of the algorithms.

(P. Maure, J. F. Aujol, and G. Peyré,2011) proposed a new adaptive framework for locally parallel texture modeling. Uses Alternating direction method Two common alternating direction methods Alternating Direction Method of Multipliers (ADMM) and Alternating Minimization Algorithm (AMA).Method used for solving PDE. Provides faster convergence than existing methods.

(B. S. He, M. Tao, and X. M. Yuan,2012) the linearly constrained separable convex programming whose objective function is separable into *m* individual convex functions with non-overlapping variables. The alternating direction method (ADM) has been well studied in the literature for the special case $m = 2$. But the convergence of extending ADM to the general case $m > 3$ is still open. In this paper, we show that the straightforward extension of ADM is valid for the general case $m>3$ if a Gaussian back substitution procedure is combined. The resulting ADM with Gaussian back substitution

is a novel approach towards the extension of ADM from $m = 2$ to $m > 3$, and its algorithmic framework is new in the literature. For the ADM with Gaussian back substitution, we prove its convergence via the analytic framework of contractive type methods and we show its numerical efficiency by some application problems

III PROPOSED METHODOLOGY

Let Target image be ‘ F ’Image decomposition is to derive ,U + V. Cartoon and Texture components are ‘ U ‘ and ‘ V‘.

Mathematically, the cartoon component can be described by a piecewise smooth (or a piecewise constant) function whilst the texture component is commonly oscillating. Because of their different properties, it is more efficient and effective to separate them for image processing and image analysis. The main task herein is to extract the cartoon and texture components from a given image with degradation, e.g., blurry and/or missing pixels.

A digital images may be sometimes distorted and degraded during image formation or transmission which could lead to a noticeable loss of visual image quality e.g. image blurring by out-of-focusing or camera shake during image acquisition; image/film deterioration due to dust spots or cracks in film; low resolution of images due to physical limit of digital camera; or noisy images caused by noisy sensors and/or transmission errors. For simplicity, if we denote images as vectors in Rn by concatenating their columns, the observed degraded version of the latent image u usually can be modeled as follows,

$$f = Hu + €$$

where f is the observed degraded image, u is the latent image, € is the image noise and H is the matrix denoting the degrading operator. The image restoration task is then to reverse the effect of the operator H on f to recover the latent image u. It is well known that image restoration is an ill-conditioned inverse problem sensitive to image noise.

In particular, these models often include a fidelity term that keeps the solutions close to the given image. By restricting the effects of the fidelity term to the complement of the inpainting region, Chan and Shen showed that very good image completions can be obtained. The principle behind their approach can be summarized as follows: variational denoising and segmentation models all have an underlying notion of what constitutes an image. In the inpainting region, the models of Chan and Shen reconstruct the missing image features by relying on this built-in notion of what constitutes a natural image.

The fidelity parameter λ in enforces the original image outside of the inpainting region. One might expect that as λ gets large, the existing region enforces some kind of effective boundary conditions on the inpainting region, these solutions turn out to approximate a solution. Results establish rigorously a connection between the inpainting technique used by Bertalmio et. al. (who prefer to impose boundary conditions at the edge of the inpainting domain D) and that of Chan et. al. (who prefer to use a fidelityterm, similar to the second term in the right hand side of our model)

A slightly modified Cahn-Hilliard equation allows us to obtain inpaintings as good as the others, but achieves them much more rapidly. This faster method is a result of both a new simplified PDE model and the use of fast solvers for such a model.

Another important feature of this new idea is that fast solvers exist for the numerical integration of the Cahn-Hilliard equation and similar diffuse interface equations. To date no such solvers have been applied to these problems in the context of imaging application that this synergistic combination of a simpler PDE based method and a state-of-the-art fast solver provides significant improvement over the previous state-of-the-art.

IMPLEMENTATION DETAILS

Cahn-Hilliard inpainting approach has many of the desirable properties of curvature based inpainting models such as the smooth continuation of level lines into the missing domain.

It provides us with a relatively simple fourth order PDE for the inpainting of binary images, rather than a more complex gradient flow to minimize a curvature functional.

Its numerical solution is an order of magnitude or more faster than other competing PDE-based inpainting methods.

The stationary solution of the limiting case Lamba tends to 0 solves.

H−1: We denote by H−1( ) the dual space of H1 (Ω) with corresponding

norm ||.||-1 . For a function f ∈ H−1( ) the norm is defined as

$$\|f\|_{-1}^2 = \left\|\nabla\Delta^{-1}f\right\|_2^2 = \int_\Omega \left(\nabla\Delta^{-1}f\right)^2\, dx.$$

By using subgradients of the TV functional within the flow, which leads to structure inpainting with smooth curvature of level sets. We motivate this new approach by a Ί−convergence result for the Cahn-Hilliard energy.

In fact we prove that the sequence of functionals for an appropriate time-discrete Cahn-Hilliard inpainting approach Ί -converges to a functional regularized with the total variation for binary arguments u = XE, where E is some Borel measurable subset of **Ω**.

This Ί −limit is generalized to an inpainting approach for grayvalue images, called TV −H−1 inpainting. There is a smooth transition layer between 0 and 1 in the Cahn-Hilliard inpainting approach (depending on the size of _) to a sharp interface limit in which the image function now jumps from 0 to 1.

Let f(~x), where ~x = (x; y), be a given image in a domain , and suppose that D _ is the inpainting domain. Let u(~x; t) evolve in time to become a fully inpainted version of f(~x) under the following equation:

$$u_t = -\Delta(\varepsilon\Delta u - \frac{1}{\varepsilon}W'(u)) + \lambda(\vec{x})(f-u)$$

$$\lambda(\vec{x}) = \begin{cases} 0 & \text{if } \vec{x} \in D, \\ \lambda_0 & \text{if } \vec{x} \in \Omega \setminus D, \end{cases}$$

The function W(u) is a nonlinear potential with wells corresponding to values of u that are taken on by most of the grayscale values. Binary images in which most of the pixels are either exactly black or white. In this binary case, W should have wells at the values

u = 0 and u = 1. W(u) = u2(u -1)2, however other functions could be used the image function u(~x; t) takes on grayscale values in a domain . and satisfies periodic boundary conditions on ∂Ω.

Alternatively, Neumann boundary conditions could be used, or any boundary conditions for which one can use fast solvers for the equation. Equation (1) is what we will call the modified Cahn-Hilliard equation, due to the added fidelity term (~x)(f - u).

The role of epsilon in equation (1) is important. In the original Cahn- Hilliard equation, serves as a measure of the transition region between two metals in an alloy, after heating and reaching a steady state. Applied to image processing, " is a measure of the transition region between the two grayscale states . for example between the black and white of printed text.

A specific fast solver known as convexity splitting is introduced. The other fast solvers might be used with good performance. Convexity splitting involves dividing up the energy functional for the equation into two parts a convex energy plus a concave energy. The part of the Euler-Lagrange equation derived from convex portion is then treated implicitly in the numerical scheme, while the portion derived from the concave part is treated explicitly.

Under the right conditions, convexity splitting for gradient flow derived equations can allow for an unconditionally gradient stable time-discretization scheme, which means arbitrarily large time steps can be taken. Vollmayr-Lee and Rutenberg have more recently refined the conditions under which stability is applicable.

The new modified Cahn-Hilliard equation is not strictly a gradient flow. The original Cahn-Hilliard equation is indeed a gradient flow using an H1 norm for the energy.

$$E_1 = \int_\Omega \frac{\varepsilon}{2}|\nabla u|^2 + \frac{1}{\varepsilon}W(u)\ d\vec{x},$$

while the fidelity term in equation can be derived from a gradient flow under an L2 norm for the energy.

$$E_2 = \lambda_0 \int_{\Omega\setminus D} (f-u)^2 d\vec{x}.$$

$$E_1 = E_{11} - E_{12}$$

Where

$$E_{11} = \int_\Omega \frac{\varepsilon}{2}|\nabla u|^2 + \frac{C_1}{2}|u|^2\ d\vec{x}$$

And

$$E_{12} = \int_\Omega -\frac{1}{\varepsilon}W(u) + \frac{C_1}{2}|u|^2\ d\vec{x}.$$

A possible splitting of E2 is

$$E_2 = E_{21} - E_{22}$$

Where

$$E_{21} = \int_{\Omega\setminus D} \frac{C_2}{2}|u|^2\ d\vec{x}$$

And

$$E_{22} = \int_{\Omega\setminus D} -\lambda_0(f-u)^2 + \frac{C_2}{2}|u|^2\ d\vec{x}.$$

The modified Cahn-Hilliard equation is neither a gradient flow in H1 nor L2. However, the idea of convexity splitting, one for the Cahn-Hilliard energy and one for the energy E2 , can still be applied to this problem with good results. The constants C1 and C2 are positive, and need to be chosen large enough so that the energies E11, E12, E21, and E22 are convex. C1 should be comparable to 1, while C2 should be comparable to 0. Numerical tests have shown that with these choices the scheme becomes unconditionally stable.

Finally the one can perform inpainting across larger regions by considering a two-step method. The inpainting is done first with a larger Epsilon, which results in topological reconnection of shapes with edges smeared by diffusion. The second step then uses the results of the first step and continues with a much smaller value of epsilon in order to sharpen the edge after reconnection. In practice such a two-stage process can result in inpainting of a stripe across a region that is over ten times the width of the stripe, without any a priori knowledge of the location of the stripe.

Expiremental Results

The modified Cahn-Hilliard equation lends itself particularly well to the inpainting of simple binary shapes, such as stripes and circles. Moreover, its applicability can be extended to achieve inpainting of objects composed of stripes and circles, i.e., roads or text.

INPUT : Blurry images, images with missing pixels, images with blurry and missing pixels.

OUTPUT : After continues iteration image undergoes inpainting and generates the original image.

IV. CONCLUSION

A coupled model for decomposing the cartoon and texture components of an image with blurry and/or missing pixels is introduced. Most of the existing papers have various limitations. Here an efficient numerical algorithm with guaranteed convergence is applied to solve the proposed model. The total variation norm and its dual norm is used to regularize the cartoon and texture. An efficient numerical algorithm based on the splitting versions of augmented Lagrangian method to solve the problem. The existence of a minimizer to the energy function and the convergence of the algorithm are guaranteed. The proposed model may not be effective when a large region of pixels are missed. For this case, an alternative strategy is to add a curvature term to the total variation term or use some nonconvex models as futher modification

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