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PSF Estimation using Checkerboard Marker

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Abstract—**A new algorithm for the estimation of centrally symmetric Gaussian point-spread function or PSF from a single blurred image is presented. A small sized, known black and white checkerboard marker is permanently installed at a corner of the camera view in the world scenario. The observed distortion of this marker in the target image is used to estimate the corresponding PSF.**

Keywords— **point-spread function, PSF, checker board marker.**

I. Introduction

Point-Spread Function is that function which convolves the input image to produce the observed image [1]. The PSF is the resulting image when the input is a point or 2d delta function. Image blur due to camera defocus or smog, *etc.* is modelled as the result of the convolution of the original unblurred image F with the corresponding Point Spread Function H. The blurred image G is represented as,

$$G = F \bullet H \qquad (1)$$

The symbol ● is used as the convolution operator. Here, G and F are matrices of same size MxN. The discrete PSF is represented by matrix H. The convolution given by Eq. (1) can also be represented by the *conv2* function as [5],

$$G = \mathrm{conv2}(F, H) \qquad (2)$$

The objective is to estimate H, given G but not F. This is the problem of estimating PSF from the single image. Several methods are available to estimate H given G [2], [3], [4].

In our proposed method, we assume that the point spread function is Gaussian. The Gaussian Point Spread Function H is centrally symmetric and is a square matrix of odd size as 3x3, 5x5, 7x7, *etc.* It is given by,

$$V(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \qquad (3)$$

Here, σ is the standard deviation of the Gaussian function. The discrete variables x and y vary in the range –m to +m. The size of V is n*n where n = (2*m+1). The elements of V are calculated using Eq. (3), for x = –m to +m and y = –m to +m. Then V is normalized to get H as,

$$H(x,y) = \frac{V(x,y)}{\text{sum of elements of V}}$$

$$= \frac{e^{-\frac{x^2+y^2}{2\sigma^2}}}{K} \qquad (4)$$

Here,K = sum of the elements of matrix V.

In terms of F(x,y) and H(x,y), the convolution is given by,

$$G(x,y) = \sum_{s=-m}^{m} \sum_{t=-m}^{m} H(s,t) * F(x-s, y-t) \qquad (5)$$

Because H is centrally symmetric, H(–s, –t) = H(s, t) and Eq. (5) can be expressed as,

$$G(x,y) = \sum_{s=-m}^{m} \sum_{t=-m}^{m} H(s,t) * F(x+s, y+t) \qquad (6)$$

The H matrix and the corresponding F matrix for a 3x3 sized H is shown [2] in Table 1 and Table 2.

Table 1. Locations of H(s,t)

H(–1,1)	H(0, 1)	H(1, 1)
H(–1,0)	H(0, 0)	H(1, 0)
H(–1, –1)	H(0, –1)	H(1, –1)

Table 2. Locations of F(x+s,y+t)

F(x–1,y +1)	F(x, y+1)	F(x+1, y+1)
F(x–1,y)	F(x, y)	F(x+1, y)
F(x –1, y–1)	F(x, y –1)	H(x+1, y –1)

From Eq. (6), we see that G(x,y) is the weighted sum of the elements of F weighed by the corresponding elements of H.

II. Assumptions and the Model

We assume that the camera is spatially fixed at a permanent location to take pictures of a predefined area. Basically, it is a surveillance set up. A black and white checker board is placed in the camera view in one of the corners of the target area as shown in Fig.1.

The checker board is placed normal to the camera view so that the image formed by it in the image plane of the camera is not distorted very much. In Fig.1, an unblurred sample image with a 2x2 checker board is shown. The size of the checker board size is properly chosen. It should be larger than the size of the expected H matrix .

Fig.1. Image with a checker board in the corner

When a blur occurs, it equally affects both the main image and the checker board. Since the original status of the checker board is well known, the blurred checker board holds the information about the blurring function H(x,y) which can be determined by examining the blurred checker board. We assume that the checker board squares are larger than the size of H so that When H is placed at the centre of a checker board square, the whole H will be within that square. Let the size of the checker board square be designated by bxb. Then b is chosen such that b is greater than (2*m+1). That is, b is greater than n.

III. THE BASIC CALCULATIONS

A. Blurred Pixel Values within the Checker Board

The blurred pixel value G(x,y) at any point (x,y) is obtained by placing the centre of the filter mask H at that point and finding the sum of the product H(s,t)*F(x+s, y+t) as given by Eq. (6). In the checker board region, F(x,y) = 0 (black) within one square region and F(x,y) = 1 (white) in the adjacent region as shown in Fig.2. Let the vertical line VV' represent the boundary between black and white regions of the checker board. The right side of VV' is white and the left side of VV' is black. For the purpose of calculation, the first white pixel column location just to the right of VV' is designated as x =0, the next one to the right as x=1 and so on as shown in Fig.2. Therefore, in the white region,

$$F(x,y) = 1 \quad \text{for } x \geq 0 \qquad (7)$$

In the black region, to the left of VV',

$$F(x,y) = 0 \quad \text{for } x < 0 \qquad (8)$$

Let the filter mask H slide from left to right across the checker board squares starting from the black region to the white region along the central line as shown in Fig.2. The value of G(x,y) is calculated along this line. Thus y remains constant at $y = y_{CL}$, where y_{CL} is the y co-ordinate of the central horizontal line.

B. Calculation of G(x,y) when H is fully on the LHS of VV'

On the LHS of VV', F(x,y) = 0 for x< 0. For a given x, y on the LHS, from Eq. (6), the range covered by F along the x axis is F(x-m, y) to F(x+m, y). The right most point of this range is (x+m). Hence, so long as (x+m)<0, the entire range (x-m) to (x+m) is less than zero and F(x+m, y) = 0 in this range. Hence G(x,y) is also zero in this region. Therefore,

$$G(x,y) = 0 \text{ for } (x+m) < 0 \qquad (9)$$

Under this condition, the filter mask H is entirely within the black region (left of VV').

When x+m = –1, the filter mark is just inside the black region as shown in Fig. 3. Here the corresponding G(x,y) remains 0. Thus,

$$G(x,y) = 0 \text{ for } (x+m) \leq -1 \qquad (10)$$

In fact, Eq. (9) and (10) are same.

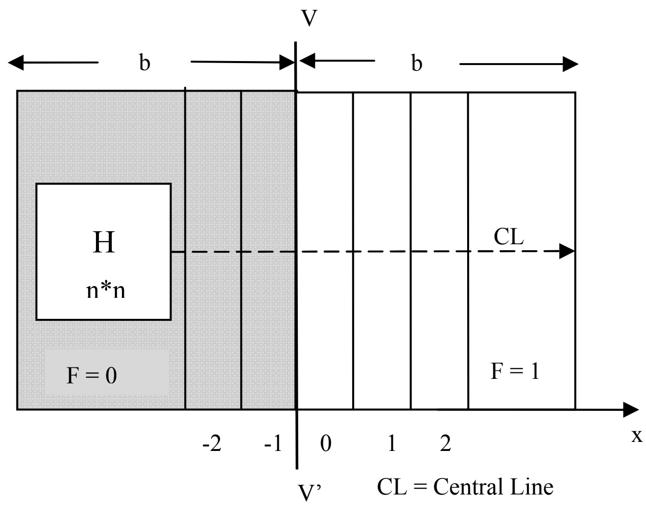

Fig.2. Black and white regions of the checker board

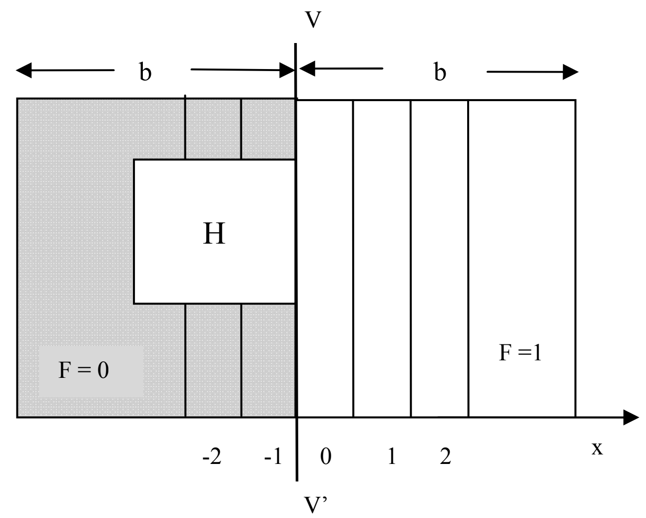

Fig.3. Entire H is just within the black region

C. Calculation of G(x,y) when H is fully on the RHS of VV'

On the RHS of VV', F(x,y) = 1 for x≥ 0. When H is on the RHS of VV', its left edge is at (x-m). So long as (x-m)≥ 0, F(x,y) = 1 in that region. That is, F(x+s,y+t) =1 for s= –m to +m and for y = –m to +m. Therefore, G(x,y) as given by Eq. (6) is,

$$G(x,y) = \sum_{s=-m}^{m} \sum_{t=-m}^{m} H(s,t) \qquad (11)$$

But, The RHS of Eq. (11) is 1, because that is the property of H(s,t). Hence,

$$G(x,y) = 1 \text{ for } (x-m) \geq 0.$$

That is, $$G(x,y) = 1 \quad \text{for } x \geq m \qquad (12)$$

When x = m, H is fully just on the RHS of VV' as shown in Fig.4.

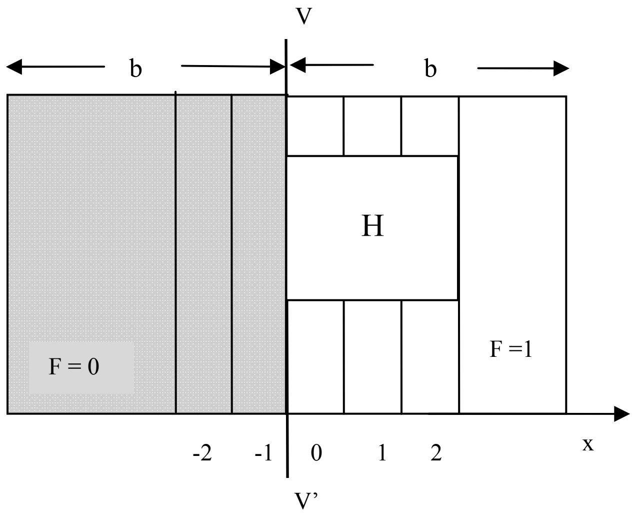

Fig.4. Entire H is just on the white region

D. Calculation of G(x,y) when H slides across VV' from Left to Right

Here x varies from (–m–1) to+ m. From Eq. (10), we see that for x= (–m–1), G(x,y) = 0. When H moves to the right of this point by one pixel to x = –m, the right most column of H lies totherightofVV' where F(x,y) =1. See Fig. 5.

At this point G(x,y) is non zero and it is calculated using Eq. (6) as,

$$G(-m,y) = \sum_{s=-m}^{m} \sum_{t=-m}^{m} H(s,t) * F(-m+s, y+t) \quad (13)$$

The above summation over s from –m to +m is split into two sub ranges as s = –m to (m–1) and s = m (to m). For s = –m to (m–1), F(–m+s, y+t) = 0 because of Eq. (8). Hence the RHS of Eq. (11) exists only for s =m. Then Eq. (11) gives,

$$G(-m,y) = \sum_{t=-m}^{m} H(m,t) * F(-m+m, y+t)$$

From Eq. (7), F(0,y+t) = 1. Hence, the above equation gives

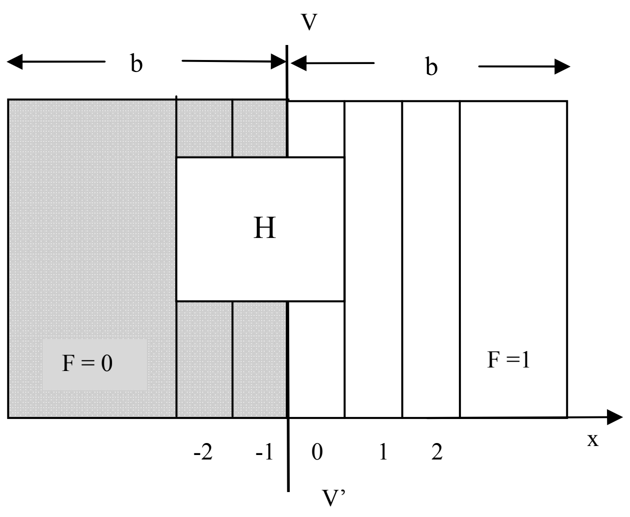

Fig.5. H has just moved by 1 column into the white region

$$G(-m,y) = \sum_{t=-m}^{m} H(m,t) \quad (14)$$

The RHS is the sum of the last column of H matrix. Because of central symmetry of H, the last column of H is same as the first column of H. Therefore, the RHS of Eq. (14) is the sum of the first column of H. Then Eq. (12) can be written as,

$$G(-m, y) = SC(1) \quad (15)$$

Here, SC(1) represents the Sum of Column 1 of H.

When x = –m+1, H has shifted by 2 columns into the RHS of VV' (white region where F = 1). Hence G(–m+1, y) is,

$$G(-m+1, y) = SC(1) + SC(2) \quad (16)$$

Similarly,

$$G(-m+2, y) = SC(1) + SC(2) + SC(3) \quad (17)$$

Similarly, in general,

$$G(-m+j, y) = SC(1) + SC(2) + \ldots + SC(j) + SC(j+1) \quad (18)$$

$$\text{where} \quad x = (-m+j) \quad (19)$$

Here, SC(j) is the Sum of Column j of H matrix. Eq. (18) holds good for j = 0 (see Fig. 5) to j =2*m (see Fig.4). When j=0, x = –m and then the first column of H is inside the F = 1 region. When j = 2*m, x=+m and H has just moved fully into the F=1 region. When j = 2*m, Eq. (18) becomes,

$$G(+m, y) = SC(1) + SC(2) + \ldots + SC(2*m) + SC(2*m+1) \quad (20)$$

The RHS of Eq. (20) gives the sum of all the columns of H which is same as the sum of matrix H which in turn is 1. Therefore, from Eq. (18) also, G(+m, y) = 1. This result is already given in Eq. (12).

In Eq. (19), (–m +j) =x, that is j = x+m. substituting these values in Eq. (18) we get,

$$G(x, y) = SC(1) + SC(2) + \ldots + SC(x+m) + SC(x+m+1) \quad (21)$$

Eqs. (10), (12) and (21) give the variation of G(x,y) within the checkerboard region. The result can be summarized as,

$G(x, y) = 0$ for $x < -m$

$G(x,y) = SC(1) + SC(2) + \ldots + SC(x+m) + SC(x+m+1)$ for $-m \le x \le +m$

$G(x, y) = 1$ for $x \ge m$

As x progressively increases from –m towards m, G(x,y) also increases progressively towards 1 as given by Eq.(21). The variation of G(x,y) as x increases from –m to m is shown in Fig.6.

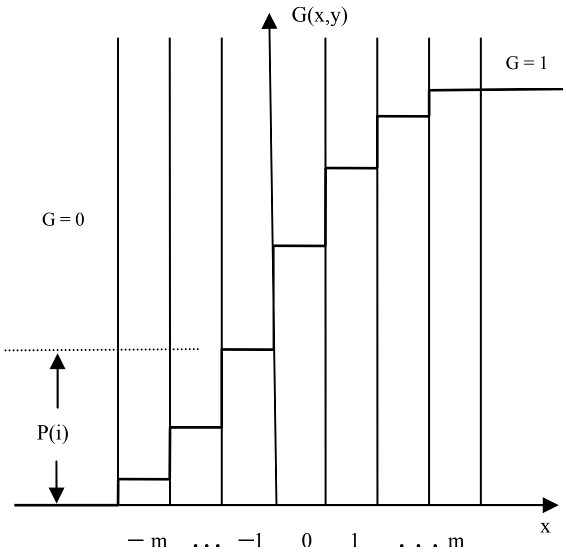

Fig. 6. Variation of G(x, y) for x = –m to +m

E. Number of increments in G(x,y)

The number of increments in G(x,y) as x varies from –m to +m can be determined from Eq. (21) as follows. For x ≤ (–m–1), the value of G(x,y) = 0 and at x = –m, it jumps to a positive value SC(1). Therefore the first increment of G(x,y) occurs at x = –m. The second increment at x = –m+1 and so on. The last increment occurs at x = m to reach 1. Therefore the successive increments (jumps) occur from x = –m to +m including both the end points. The number of points within this range is, from –m to –1 on the LHS of VV' and from 0 to m on the RHS of VV'. The number of increments to the left of VV' is m and to the right of VV' is (m+1). The total number of increments, designated by T is,

T = m+(m+1)=2*m +1 (22)

Therefore the size n of H is same as the number of increments in G(x,y) for x = (–m–1) to +m. that is,

n =T=2*m +1 (23)

F. Determination of n, the size of H

The pixel values G(x,y) is scanned along the Central Line (see Fig, 2) from x = – b/2 to + b/2 and stored in array g. This range – b/2 to + b/2 includes the range (–m–1) to +m. Therefore, The array g includes the successively increasing values of G(x,y) from 0 to 1 (see Fig.6). The number of successive increments in the values of array g is determined as follows.

Find the first difference of g designated by q as,

q(i) = diff(g) = g(i+1) –g(i) (24)

for i = 1 to length(g).

Count the number of non zero elements of array q to get T which is same as n. Then m =(n–1)/2.

G. Determination of σ, the standard deviation of H

Now, we know n and m. The first non zero element of array g gives G(–m, y). This is equal to SC(1) from Eq. (16). Let us call it p(1). That is,

p(1) = first non zero element of g = G(–m, y) = SC(1)

similarly, from Eq. (17),

p(2) = second non zero element of g = G(–m+1, y)
= SC(1)+SC(2)

……………………………………………………….

p(i) = i th non zero element og g =
= SC(1) +SC(2)+….+SC(i-1)+SC(i) (25)

…………………………………………………..

p(n) = n th non zero element og g = G(m, y)
= SC(1) +SC(2)+….+SC(n-1)+SC(n)=1

Eq. (25) holds good for i = 1,2, …,n.

Now consider the difference

u(i) = p(i) –p(i–1) (26)

for i=1,2,…,n. Take p(0) = 0.

From Eq. (25), we see that

u(i) = SC(i) (27)

for i=1 to n.

H has a total of 2*m+1 columns where n = 2*m+1. The (m+1) th column of H is its central column. Because of Gaussian elements, the sum of central column, SC(m+1) is maximum among all the column sums. The central column of H corresponds to H(x, y) with x = 0 and y varying from –m to m. From the definition of H(x,y) as defined by Eqs. (3) and (4) the sum of the central column of H(x, y) is found as follows.

H. Sum of the Central and next column of matrix H(x,y)

For the central column, x = 0 and y varies from –m to m. From Eq. (4), the sum of the central column of H(x, y) is

$$\mathrm{SC}(m+1) = \sum_{y=-m}^{m} H(0,y) = \frac{1}{K}\sum_{y=-m}^{m} e^{-\frac{y^2}{2\sigma^2}} \qquad (28)$$

For the next column of H, x =1 and y varies from –m to m. From Eq. (4), the sum of the column (m+2) of H(x, y) is,

$$\mathrm{SC}(m+2) = \sum_{y=-m}^{m} H(1,y) = \frac{1}{K}\sum_{y=-m}^{m} e^{-\frac{1^2+y^2}{2\sigma^2}} \qquad (29)$$

Eq.(29) can be rewritten as,

$$\mathrm{SC}(m+2) = \frac{1}{K}\sum_{y=-m}^{m}\left(e^{-\frac{1^2}{2\sigma^2}}\right) * \left(e^{-\frac{y^2}{2\sigma^2}}\right) \qquad (30)$$

I. Expression for σ

In Eq. (30), $\left(e^{-\frac{1^2}{2\sigma^2}}\right)$ is independent of y and can be taken outside the ∑ and Eq. (30) can be rewritten as,

$$\mathrm{SC}(m+2) = \frac{1}{K} * \left(e^{-\frac{1^2}{2\sigma^2}}\right) * \sum_{y=-m}^{m}\left(e^{-\frac{y^2}{2\sigma^2}}\right) \qquad (31)$$

Dividing Eq.(28) by Eq. (31) gives,

$$\frac{\mathrm{SC}(m+1)}{\mathrm{SC}(m+2)} = e^{\frac{1^2}{2\sigma^2}} \qquad (32)$$

Taking natural log on both sides, we get

$$\frac{1}{2*\sigma^2} = \mathrm{Log}\left(\frac{\mathrm{SC}(m+1)}{\mathrm{SC}(m+2)}\right) \qquad (33)$$

Solving for σ, we get,

$$\sigma = \sqrt{\frac{1}{2 * \text{Log}\left[\frac{\text{SC}(m+1)}{\text{SC}(m+2)}\right]}} \quad (34)$$

J. Algorithm to find n and σ

Algorithm 1. Input: G(x,y) values along the central line of the checkerboard region. Output: n and σ of the Gaussian filter H.

1. Scan G(x,y) along the Central Line (see Fig, 2) from x = – b/2 to + b/2 and store the result in array g.
2. Find the first difference of g designated by q as,
   q(i) = diff(g) = g(i+1) –g(i) for i=1 to length(g).
3. Count the non zero elements of q to get n.
4. Set m = (n–1)/2.
5. Get p(i) as the i th non zero element of g for i=1 to n.
6. Take p(0)=0 and get SC(i) as,
   SC(i) = p(i) – p(i–1) for i=1 to n.
7. Get SC(m+1) and SC(m+2).
8. Determine σ from Eq.(34).

Then H is determined using Eq. (3) and (4).

IV. Experimental results

Example 1. In the simulated experiment, un-blurred image Lena is used with a super imposed 2x2 checkerboard as shown in Fig. 1. The size of the image is 512x512 and the size of the checkerboard square is 60x60. The value of b is 60. The un-blurred image with checker board is blurred using a Gaussian PSF given by H as [3],

H = fspecial('gaussian', [7 7] , 2.3).

Here the size of H is 7x7. That is, n = 7 and m =3. The standard deviation used in H is, σ = 2.3. The elements of H matrix are shown in Table 3. The blurring is carried out by convolving the image matrix with H. From the blurred image array g is found. From g, after finding diff(g) the value of n is found. This n is found to be exactly 7. The p array is found from g as,

p = [0.0847 0.2205 0.4009 0.5991 0.7795 0.9153 1.0000]

The SC array is found to be,

SC = [0.0847 0.1358 0.1804 0.1982 0.1804 0.1358 0.0847]

SC(m+1) = SC(4) = 0.1982.

SC(m+2) = SC(5) = 0.1804;

The ratio SC(m+1) /SC(m+2) = 1.0991.

Then σ is found as,

$$\sigma = \sqrt{\frac{1}{2 * \text{Log}[1.0991]}} = 2.300$$

Thus the assumed values n and σ are determined exactly.

The variation of G(x,y) across the checkerboard region is shown in Fig.7 in pseudo colors.

The experiment was repeated for different n's and σ's and the equations gave correct results.

Example 2. Here n = 11 and σ = 5. The value of G(x,y) for x= – b/2 tp +b/2 along the central line stored in array g is shown in Fig. 8. The increments in g from x = – 5 to + 5 can be seen in fig.8.

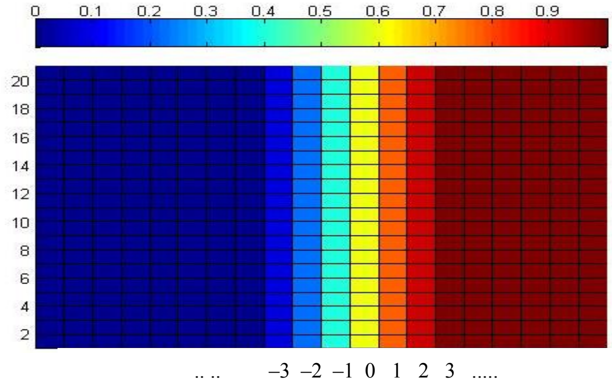

Fig. 7. Variation of G(x,y) across the checker board region

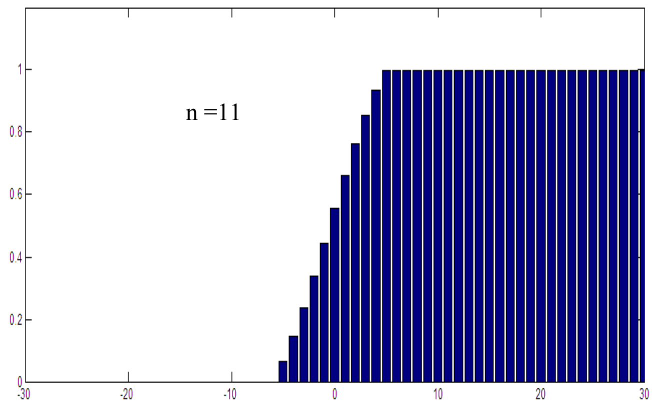

Fig. 8. Variation of G(x,y) as x varies from –b/2 to +b/2 along the central line of checkerboard.

V. Conclusions

A new method of estimating the Gaussian PSF is presented. The method determines the size and σ of the Gaussian PSF accurately.

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