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Burst Noise Detection and Cancellation in DS-CDMA

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Abstract— **This paper presents a new algorithm for the detection and correction of certain type of burst noise spikes in DS-CDMA receivers.**

Keywords— **DS-CDMA, burst noise cancellation, Walsh – Hadmard orthogonal codes.**

I. INTRODUCTION

Code division multiple access (CDMA) is a channel access method which allows multiple users to transmit the data at the same time. At present CDMA is extensively used in mobile phones, wireless LANs etc.[1]. In the DS-CDMA system, the user data is multiplied by a code sequence. This code sequence is also called the signature or chip. Signatures are unique for each user and they identify the user at the receiving side. Several types of signature sequences like pseudo-noise sequences, gold sequences etc. are in usage for DS-CDMA. Here, we use Walsh–Hadmard (W–H) orthogonal codes (chips or Signatures) for encoding and decoding data bits.

Burst noise or impulse noise is a series of intermittent noise pulses occurring at random intervals. It can have positive or negative random magnitude values. It can occur due to electrical disturbances or interferences from other users (MAI) and jammers.

Several techniques are used to reduce the general noise and the burst noise effects in CDMA system [2], [3], [4]. But our method is different from them.

Our goal is to detect and cancel certain type of burst noise spikes at the DS-CDMA receiver.

II. BASIC MODEL, TERMS AND DEFINITION

A. Basic DS-CDMA Model

The basic model is shown in Fig.1. The user data inputs are $b_1, b_2, ..., b_K$ where K is the number of users or the number of channels of the DS-CDMA. $S_1, S_2, ..., S_K$ are the signatures of the corresponding users. The length of each signature is also taken as K. Therefore the spreading or processing gain is K. We use the Walsh–Hadmard (W–H) orthogonal codes as the signature. The W–H codes are obtained as follows.

The 2x2 Hadamard matrix is given by,

$$H_2 = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix} \qquad (1)$$

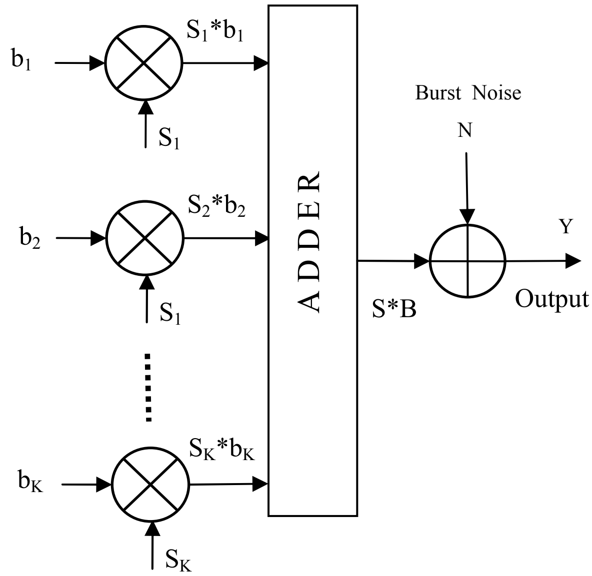

Fig.1. Basic DS-CDMA Model

The successive H's are obtained by the recursion

$$H_{2m} = \begin{bmatrix} +H_m & +H_m \\ +H_m & -H_m \end{bmatrix} \text{ for } m = 2, 4, 8, 16... \qquad (2)$$

For, m = 2, H_4 is given as

$$H_4 = \begin{bmatrix} +H_2 & +H_2 \\ +H_2 & -H_2 \end{bmatrix} = \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix}$$

From H_4, H_8 is obtained as,

$$H_8 = \begin{bmatrix} +H_4 & +H_4 \\ +H_4 & -H_4 \end{bmatrix}$$

At present, we take K = number of users = 8. Here, the 8 signatures are obtained from the columns of H_8 in the order 1 to 8. Therefore the signature matrix S is,

$$S = H_8 = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 \end{bmatrix} \quad (3)$$

S is symmetric, that is $S^T = S$. It is also orthogonal, that is,

$$S^T * S = 8 * I \quad (4)$$

where I is the identity matrix of size 8x8. It means,

$$\left.\begin{aligned} S_i^T * S_j &= 8 \text{ if } j = i \\ S_i^T * S_j &= 0 \text{ if } j \neq i \end{aligned}\right\} \quad (5)$$

The signature S_i of user i is the column i of H_8 for i = 1 to 8. The length of S_i is also 8. The elements of the column vector S_i represent the individual spreading values. The data bit b_i is spread as,

$$D_i = S_i * b_i = [\, s_{1i} * b_i \quad s_{2i} * b_i \ \ldots \ s_{8i} * b_i \,]^T \quad (6)$$

with $S_i = [s_{1i} \quad s_{2i} \ \ldots \ s_{8i}]^T$ for I = 1 to 8.

The spreading action is shown in Fig.2, for
$S_6 = [\, +1 \ -1 \ +1 \ -1 \ -1 \ +1 \ -1 \ +1 \,]$

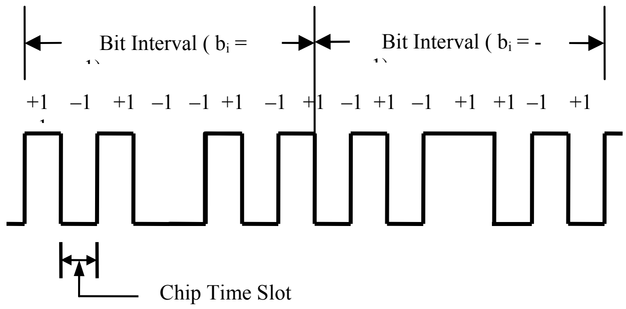

Fig.2. Spreading of b_i by S_6 for b_i = 1 and b_i =

The individual D_i's are added (see Fig.1) to get the output of the Adder of the model DS-CDMA transmitter system as,

$$\sum_{i=1}^{8} D_i = \sum_{i=1}^{8} S_i * b_i = [S_1 \quad S_2 \ \ldots \ S_8] * \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_8 \end{bmatrix} = S * B$$

where, S is the signature matrix as given by Eq. (3) and

B is the data vector containing individual data bits as,

$$B = [b_1 \ b_2 \ \ldots \ b_8]^T \quad (7)$$

After adding the Noise vector N to this S*B we get the output of DS-CDMA system, we get [5],

$$Y = S * B + N \quad (8)$$

The burst noise is represented by the column vector of length 8 as,

$$N = [\, n_1 \quad n_2 \ \ldots \ n_8 \,]^T \quad (9)$$

where n_i is the noise spike at chip time slot i, for i =1 to 8.

In Eq. (8) the output Y is a vector of length 8 as,

$$Y = [\, y_1 \quad y_2 \ \ldots \ y_8 \,]^T \quad (10)$$

Here, y_i is the value of the output at chip time slot i, for i =1 to 8 and y_i is given by

$$y_i = \sum_{j=1}^{8} s_{ij} * b_j + n_i \quad (11)$$

for i = 1 to 8.

Eq. (8) is a vector-matrix equation the sizes of Y, B, N and S are as follows,
Sizes of Y, B and N are all 8x1, that is 8 rows and 1 column.
Size of S is 8x8, that is 8 rows and 8 columns.

In our DS-CDMA model the modulation (bpsk/qpsk) and the corresponding synchronous demodulation used in actual DS-CDMA are not taken into consideration. We assume that the demodulation process exactly recovers the base band data without distortion. Also we assume the multipath fading factor as 1 for all channels and only burst noise is present.

III. Basic Principle of Noise Spike Cancellation

A. *Data Carrying and non Data Carrying Channels*

Among the 8 available channels, only 4, 6, 7 and 8 carry data. That is b_4, b_6, b_7 and b_8 carry the actual user data. The user data belongs to $\{-1, +1\}$. The data inputs of channels 1, 2, 3 and 5 are kept zero throughout so that the information received on those channels represent only noise information. From this information noise spike values are determined. In the case of 8 users, we classify the data into two disjoint groups as,

$$B_1 = [\, b_1 \quad b_2 \quad b_3 \quad b_5]^T \quad (12)$$

$$B_2 = [\, b_4 \quad b_6 \quad b_7 \quad b_8]^T \quad (13)$$

In our scheme,

$$b_1 = b_2 = b_3 = b_5 = 0 \quad (14)$$

That is,

$$B_1 = [\, 0 \ 0 \ 0 \ 0]^T \quad (15)$$

The elements of data carrying group B_2 are ±1. Therefore,

$$abs(B_2) = [1 \ 1 \ 1 \ 1]^T \quad (16)$$

Eqs. (15) and (16) are the characteristics of the data in our scheme. Eqs. (15) and (16) are also conveniently written as,

$$B_1 = 0 \quad (17)$$

and

$$abs(B_2) = 1 \quad (18)$$

Here, B_1 is a 4x1 vector and the 0 on the RHS of Eq. (17) represents a 4x1 all zero vector. Similarly, abs(B_2) is a 4x1 vector and the 1 on the RHS of Eq. (18) represents a 4x1 vector of all 1's. In our scheme, Eqs. (15) and (16) together represent the most important characteristics of the data channel.

Therefore, in our scheme, 50% of the data channels are reserved for spike noise detection and cancellation. The channels 1, 2, 3 and 5 are specifically chosen as non data carrying channels, because the corresponding signatures S_1, S_2, S_3 and S_5 form a special pattern as shown below.

$$[S_1 \quad S_2 \quad S_3 \quad S_5] = \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & -1 & -1 & +1 \\ +1 & +1 & +1 & -1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & -1 \end{bmatrix} \qquad (19)$$

Let $R = [\, S_1 \ S_2 \ S_3 \ S_5 \,]^T$ (20)

Then taking the transpose of the RHS of Eq. (20) we get,

$$R = \begin{bmatrix} S_1^T \\ S_2^T \\ S_3^T \\ S_5^T \end{bmatrix} \qquad (21)$$

From Eqs. (19) and (21),

$$R = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \end{bmatrix} \qquad (22)$$

Here, row 1 has all 1's with no sign change, row 2 has alternating 1's and -1's 1 at a time, row 3, 2 at a time and row 4, 4 at a time. rows 2, 3 and 4 hold all possible binary combinations of 3 bit word.

B. Separation of Data and non Data channels

Consider the decorrelation detection for Eq. (8). Multiply both sides of Eq.(8) by S^T and let $Z = S^T*Y$, then

$$Z = S^T*Y = S^T*S*B+S^T*N \qquad (23)$$

Since $S^T*S = 8*I$, the above equation can be rewritten as,

$$Z = S^T*Y = 8*B+S^T*N \qquad (24)$$

Here, Z is a column vector of size 8 as,

$$Z = [\, z_1 \ z_2 \ldots z_8 \,]$$

Eq. (18) represents 8 equations as,

$$z_i = S_i^T*Y = 8*b_i + S_i^T*N \qquad (25)$$

Here, z_i is the i th element of Z.

Now, z_i's are grouped into two disjoint groups as,

$$U = [\, z_1 \ z_2 \ z_3 \ z_5 \,]^T \qquad (26)$$

$$V = [\, z_4 \ z_6 \ z_7 \ z_8 \,]^T \qquad (27)$$

U contains elements 1, 2, 3 and 5 of Z and V contains remaining elements 4, 6, 7 and 8 of Z. On substituting for z_1 z_2 z_3 and z_5 in Eq. (26) from Eq. (25) we get,

$$U = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_5 \end{bmatrix} = 8 * \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_5 \end{bmatrix} + \begin{bmatrix} S_1^T \\ S_2^T \\ S_3^T \\ S_5^T \end{bmatrix} * N \qquad (28)$$

Since $b_1 = b_2 = b_3 = b_5 = 0$, (see Eq. (14)), Eq.(28) becomes,

$$U = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_5 \end{bmatrix} = \begin{bmatrix} S_1^T \\ S_2^T \\ S_3^T \\ S_5^T \end{bmatrix} * N \qquad (29)$$

In Eq. (29), signature vectors are substituted from Eq. (21) to get,

$$U = R*N \qquad (30)$$

Substituting for R from Eq. (22) and N from Eq. (9),

$$U = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \end{bmatrix} * \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \\ n_8 \end{bmatrix} \qquad (31)$$

Eq. (31) is the most important equation for our method. Since U is known, Eq. (31) represents 4 linear equations in 8 unknown variables n_1 to n_8. Under certain circumstances, Eq. (31) can be solved for some of these noise elements n_1 to n_8.

From Eqs. (27) and (25) we can express V as,

$$V = \begin{bmatrix} z_4 \\ z_6 \\ z_7 \\ z_8 \end{bmatrix} = 8 * \begin{bmatrix} b_4 \\ b_6 \\ b_7 \\ b_8 \end{bmatrix} + \begin{bmatrix} S_4^T \\ S_6^T \\ S_7^T \\ S_8^T \end{bmatrix} * N \qquad (32)$$

Here, $B_2 = [\, b_4 \ b_6 \ b_7 \ b_8 \,]^T$ is the actual user data set and the members of this data set can take values -1 or +1 only. This fact is also used in determining the noise element values.

IV. ZERO NOISE DETECTION

Let $N = [\, 0 \ 0 \ \ldots \ 0]^T$. That is, $n_i = 0$ for i=1 to 8. In this case, from Eq. (31), $U = [0 \ 0 \ 0 \ 0\,]^T$. Also from Eq. (32),

$$V = 8* [\, b_4 \ b_6 \ b_7 \ b_8 \,]^T = 8*B_2 \qquad (33)$$

Since, $abs(b_i) = 1$ for i = 4, 6, 7 and 8, taking absolute values of both sides of Eq. (33) gives,

$$abs(V) = [\, 8 \ 8 \ 8 \ 8 \,]^T = 8*abs(B_2) \qquad (34)$$

Hence, conditions $U = [\, 0 \ 0 \ 0 \ 0 \,]^T$ and $abs(V) = [8 \ 8 \ 8 \ 8]^T$ imply $N = [\, 0 \ 0 \ \ldots 0]^T$.

V. SINGLE NOISE SPIKE DETECTION

Let the noise vector contain a single spike at location j as,

$$N = [\, 0 \ldots 0 \quad n_j \ \ 0 \ldots 0]^T \qquad (35)$$

That is, n_i is assumed to be zeros for i = 1 to 8 except at certain single j where $1 \le j \le 8$. Here, our object is to determine the value of j and n_j.

A. Determination of Noise Location and value

Consider Eq. (30). It can be rewritten as,

$$U = [R_1\ R_2 \dots R_8] * N = [R_1\ R_2 \dots\ R_8] * \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \\ n_8 \end{bmatrix} = \sum_{i=1}^{8} R_i * n_i \qquad (36)$$

Here, $R_1, R_2, \dots, R_8$ are the columns of matrix R. See Eq. (22).

Since n_i exists only at i = j, the above summation reduces to,

$$U = R_j * n_j \qquad (37)$$

Let us consider two cases when n_j is +ve and when it is –ve.

1) n_j is + ve: In this case sign(R_j*n_j) is same as sign(R_j). Taking the sign on both sides of Eq. (36),

$$\text{sign}(U) = \text{sign}(R_j) \qquad (38)$$

Since R_j contains only ±1's, sign(R_j) = R_j (39)

From Eqs. (39) and (38),

$$R_j = \text{sign}(U) \qquad (40)$$

Substituting for U from Eq. (26), Eq. (40) represents 4 equations as,

$$\begin{bmatrix} R_{1j} \\ R_{2j} \\ R_{3j} \\ R_{4j} \end{bmatrix} = \begin{bmatrix} \text{sign}(z_1) \\ \text{sign}(z_2) \\ \text{sign}(z_3) \\ \text{sign}(z_5) \end{bmatrix} \qquad (41)$$

Where R_{1j}, R_{2j}, R_{3j} and R_{4j} are the elements of column R_j.

2) n_j is – ve: In this case sign(R_j*n_j) is –sign(R_j). Taking the sign on both sides of Eq. (37),

$$\text{sign}(U) = -\text{sign}(R_j)$$

Since R_j contains only ±1's, sign(R_j) = R_j and the above equation becomes

$$-R_j = \text{sign}(U) \qquad (42)$$

This again can be written expanded as,

$$\begin{bmatrix} -R_{1j} \\ -R_{2j} \\ -R_{3j} \\ -R_{4j} \end{bmatrix} = \begin{bmatrix} \text{sign}(z_1) \\ \text{sign}(z_2) \\ \text{sign}(z_3) \\ \text{sign}(z_5) \end{bmatrix} \qquad (43)$$

Therefore, in the case of single spike noise, either Eq. (40)) or (42) is satisfied.

3) To determine the location j of the noise spike: The location of the noise spike is j. It is also the column number which satisfies Eq. (40) or (43). R_j should be one of the columns of R. To find j, we scan the columns of R successively starting from 1 towards 8, until Eq. (40) or (42) is satisfied.

Starting from column 1, compare the columns of R and –R to sign(U) until the match is found. Then the matching column number gives the value of j. This process can be written as

Algorithm 1. INPUT : U and R. OUTPUT : j.

```
for i = 1 to 8
    if ( Ri = = sign(U) ) or (Ri = = –sign(U) )
        j = i    (store this specific i)
        break   (come out of the for loop)
    end if
end for
```

4) To determine the value of n_j; From Eq. (26), we know that U is a 4x1 vector. Therefore Eq. (30) represents 4 equations as,

$$U = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_5 \end{bmatrix} = R_j * n_j = \begin{bmatrix} R_{1j} * n_j \\ R_{2j} * n_j \\ R_{3j} * n_j \\ R_{4j} * n_j \end{bmatrix}$$

Consider the equation corresponding to the first row,

$$z_1 = R_{1j} * n_j$$

From this and knowing that R_{1j}=1, we calculate n_j as,

$$n_j = z_1 \qquad (44)$$

B. Cancellation of the Noise spike

Once the location (j value) and the value of n_j are known, the noise vector N is reconstructed as given by Eq. (35). This N is subtracted from Y of Eq. (8) to cancel N and to get,

$$Y–N = S*B+N–N = S*B \qquad (45)$$

Pre-multiplying both sides by S^T yields Data vector B as,

$$S^T*(Y–N) = S^T*S*B = 8*B$$

Then, $B = (1/8)*S^T*(Y–N)$ (46)

B is an 8x1 vector. Separate the calculated B into two disjoint groups as in Eqs. (12) and (13) to get $B_1 = [\ b_1\ \ b_2\ \ b_3\ \ b_5]^T$ and,

$$B_2 = \begin{bmatrix} b_4 \\ b_6 \\ b_7 \\ b_8 \end{bmatrix} \qquad (47)$$

Now, $B_1 = 0$ and abs(B_2) = $[1\ \ 1\ \ 1\ \ 1]^T$ assure us that the noise spike detection and correction is correct. Eq.(47) gives the correct data after cancellation.

VI. DOUBLE NOISE SPIKE DETECTION AND CANCELLATION

Let the noise vector N contain two spikes at locations j and k for $0 \le j < k \le 8$. Then the noise vector appears as,

$$N = [\ 0\ .\ \ n_j\ .\ 0\ .\ n_k\ .\ 0]^T \qquad (48)$$

Here, two of the elements at locations j and k in N are non zeros and the remaining elements are zeros. Here j is the lower index and k is the higher index. Our objective is to determine the distinct locations j, k and the values n_j and n_k.

A. *Determination of Noise Locations and values*

Substituting for N in Eq. (36) from Eq. (48), we get,

$$U = R_j*n_j + R_k*n_k \quad (49)$$

This can be expressed in the matrix form as,

$$U = [R_j \quad R_k] * \begin{bmatrix} n_j \\ n_k \end{bmatrix} \quad (50)$$

Here, U is a 4x1 vector of known values. R_j and R_k are the columns j and k of R. That is, $[R_j \quad R_k]$ is a matrix of size 4x2. Eq. (50) represents 4 equations in two variables n_j and n_k. It is a over determined system of equations.

1) Example 1: Let j = 2 and k = 7 with $n_2 = n_7 = 1$. From Eq. (49), $U = R_2+R_7$. Substituting for columns R_2 and R_7 from Eq. (22), $U = [\,2\ 0\ 0\ 0\,]^T$. Now let j = 3 and k = 6 with $n_3 = n_6 = 1$. Then also $U = R_3+R_6 = [\,2\ 0\ 0\ 0\,]^T$. Thus $U = [\,2\ 0\ 0\ 0\,]^T$ has multiple solutions for set [j k] which satisfies Eq. (49) or Eq. (50). (For convenience, the set is represented by elements enclosed within square brackets). In fact, for this vector $U = [\,2\ 0\ 0\ 0\,]^T$, there are four solutions for set [j k] as,

[j k] = { [1 8], [2 7], [3 6], [4 5] }.

This example shows that Eq. (50) or (49) can have more than one set of solutions for [j k]. When there is more than one solution set, only one of them is valid. Determination of this valid set based on Eqs. (15) and (16) will be described later.

2) Solving Equation (50) : From Eq. (22) for R, we can see that any two distinct columns of R are linearly independent. Hence the rank of $[R_j \quad R_k]$ is 2. Therefore Eq. (50) can be solved as follows.

$$\text{Let} \quad P(j,k) = [R_j \quad R_k] \quad (51)$$

The size of P(I, k) is 4x2. Then Eq. (50) becomes,

$$U = P(j,k) * \begin{bmatrix} n_j \\ n_k \end{bmatrix} \quad (52)$$

Pre multiplying both sides by $P(j, k)^T$, we get

$$P(j,k)^T * U = P(j,k)^T * P(j,k) * \begin{bmatrix} n_j \\ n_k \end{bmatrix}$$

Pre multiplying both sides by $[P(j,k)^T * P(j,k)]^{-1}$ gives,

$$\begin{bmatrix} n_j \\ n_k \end{bmatrix} = [P(j,k)^T * P(j,k)]^{-1} * P(j,k)^T * U \quad (53)$$

Note that the matrix inverse of Eq. (53) exists because the rank of P(j, k) is 2 and the size of P(j, k) is 4x2. Now, substitute for $\begin{bmatrix} n_j \\ n_k \end{bmatrix}$ in Eq. (52), from Eq. (53) to get,

$$U = P(j,k) * [P(j,k)^T * P(j,k)]^{-1} * P(j,k)^T * U \quad (54)$$

Let

$$Q(j,k) = P(j,k) * [P(j,k)^T * P(j,k)]^{-1} * P(j,k)^T \quad (55)$$

Then Eq. (54) can be expressed as,

$$U = Q(j, k)*U \quad (56)$$

B. *Determination of correct j, k, n_j and n_k*

Indices j and k should be so chosen that the matrix P(j, k) should satisfy Eq. (54) where P(j, k) is defined by Eq. (51). As j and k vary, P(j, k) varies and for certain values of j and k, Eq. (54) is satisfied. We start with j = 1, k = 2 and go over all possible distinct combinations of j and k to satisfy Eq. (54). The range of j as well as k is 1 to 8 and k > j. More than one solution may exist for [j k]. The correct solution is determined as follows.

Using the present set of j, k, n_j and n_k, construct the current noise vector as indicated in Eq. (48). Call this as N_c. Subscript c designates that N_c is the present calculated noise vector, but need not be the final correct N. Then determine the calculated data vector B_c (calculated) using Eq. (46). From B_c, we find B_{1c} and B_{2c} as indicated in Eqs. (12) and (13). Then, conditions given by Eqs. (15) and (16) are used to validate the correctness of the solution as,

$$\left.\begin{aligned} B_{1c} &= [0 \quad 0 \quad 0 \quad 0]^T \\ abs(B_{2c}) &= [1 \quad 1 \quad 1 \quad 1]^T \end{aligned}\right\} \quad (57)$$

If Eq.(57) is satisfied, it means the present set of j, k, n_j and n_k, are the correct solutions. B_c and N_c are the correct data and noise vectors.

The algorithm to find correct j and k along with n_j and n_k is given as follows. **Algorithm 2.**

```
for j = 1 to 7
  for k = j+1 to 8
    get P(j, k) as P(j, k) = [Rj  Rk].
     check whether this P(j, k) satisfies Eq. (54) or not.
          If P(j, k) satisfies Eq. (54),
             Use this j and k and
              get [nj nk ]^T by Eq. (53).
              Knowing j, k, nj and nk
        construct the noise vector Nc as indicated by Eq. (48).
              Calculate Bc similar to Eq.(46) as,
                 Bc = (1/8)*S^T*(Y–Nc )
              Get B1c and B2c from Bc
                   as indicated by Eqs. (12) and (13).
              Check whether Eq. (57) holds true .
               If true,
                    this Nc is and Bc are the correct answers.
                    That is, B = Bc and N = Nc . Exit.
               end if
            end if
               // go to next j, k iterations
      end for   // for k
  end for    // for j
```

At the end of this algorithm, $N = N_c$ gives the correct noise vector and $B = B_c$ gives the correct data vector.

C. *Example 2*

Let $N=[0\ 0\ 0\ 5\ 5\ 0\ 0\ 0]^T$ and

$B = [\,0\ 0\ 0\ 1\ 0\ -1\ 1\ 1]^T$.

From Eq. (8), $Y = S*B+N = [2\ 0\ -4\ 7\ 5\ -2\ 2\ 0]^T$.

From Eq. (24) $Z = S^T*Y = [\,10\ 0\ 0\ 18\ 0\ -18\ -2\ 8\,]^T$.

From Eqs.(26) and (27), U and V are given by,

$U = [10\ 0\ 0\ 0\,]^T$ and $V = [\,18\ -18\ -2\ 8\,]^T$. In this example, 4 set of [j k] values satisfy Eq. (54). These set are,

[j k] = { [1 8], [2 7], [3 6], [4 5] }.

Consider each set at a time. When [j k] = [1 8], from Eqs. (51) and (22), P(1, 8) is expressed in terms of the column vectors R_1 and R_2 as,

$$P(1,8) = [\,R_1\;\; R_8\,] = \begin{bmatrix} +1 & +1 \\ +1 & -1 \\ +1 & -1 \\ +1 & -1 \end{bmatrix}$$

Corresponding Q(1, 8) from Eq. (55) is given by,

$$Q(1,8) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.33 & 0.33 & 0.33 \\ 0 & 0.33 & 0.33 & 0.33 \\ 0 & 0.33 & 0.33 & 0.33 \end{bmatrix}$$

This Q(1, 8) satisfies Eq. (56). (That is Eq. (54) is satisfied. From Eq. (53), $[\,n_1\;\; n_8]^T$ is found to be $[\,n_1\;\; n_8]^T = [\,5\;\; 5]^T$.

Now, the noise vector N_c would be

$N_c = [\,5\;\; 0\;\; 0\;\; 0\;\; 0\;\; 0\;\; 0\;\; 5]^T$.

The value of B_c as in Eq. (46) is given by,

$B_c = [\,0\;\; 0\;\; 0\;\; 1\;\; 0\;\; -3.5\;\; -1.5\;\; 1\,]^T$.

From this, as in Eqs.(12) and (13),

$B_{1c} = [\,0\;\; 0\;\; 0\;\; 0\,]^T$ and $B_{2c} = [\,1\;\; -3.5\;\; -1.5\;\; 1\,]^T$. Now, we see that the condition of Eq. (57), viz $abs(B_{2c}) = [\,1\;\; 1\;\; 1\;\; 1]^T$ is not satisfied. Therefore [j k] = [1 8] is not the correct solution. Similarly, [j k] = [2 7] and [3 6] are found to be incorrect. The last set [j k] = [4 5] satisfies all the conditions and gives the correct noise vector $N = [0\;\; 0\;\; 0\; +5\; +5\;\; 0\;\; 0\;\; 0]^T$.

VII. DETECTION AND CANCELLATION THREE NOISE SPIKES

Let j, k and m be the locations of noise spikes in N. That is, $n_i = 0$ for $i \notin [\,j\;\; k\;\; m\,]$ and $n_i \neq 0$ for $i \in \{\,j\;\; k\;\; m\,\}$. Also we take $1 \leq j < k < m \leq 8$. Under this condition, Eq. (36) gives,

$$U = R_j * n_j + R_k * n_k + R_m * n_m = [R_j \quad R_k \quad R_m] * \begin{bmatrix} n_j \\ n_k \\ n_m \end{bmatrix} \qquad (58)$$

Let $P(j, k, m) = [R_j\;\; R_k\;\; R_m]$ (59)

Then Eq. (58) becomes,

$$U = P(j,k,m) * \begin{bmatrix} n_j \\ n_k \\ n_m \end{bmatrix} \qquad (60)$$

P(j, k, m) is a 4x3 matrix comprising 3 distinct columns of R. By observing R in Eq. (22), we see that any 3 distinct columns of R are linearly independent. Hence the rank(P(j, k, m)) = 3. Hence, Eq. (60) can be solved for $[n_j\;\; n_k\;\; n_m]^T$ as,

$$\begin{bmatrix} n_j \\ n_k \\ n_m \end{bmatrix} = [P(j,k,m)^T * P(j,k,m)]^{-1} * P(j,k,m)^T * U \qquad (61)$$

Pre multiplying the above equation by P(j, k, m), we get,

$$P(j,k,m) * \begin{bmatrix} n_j \\ n_k \\ n_m \end{bmatrix} =$$

$$= P(j,k,m) * [P(j,k,m)^T * P(j,k,m)]^{-1} * P(j,k,m)^T * U \quad (62)$$

Let Q(j, k, m) =

$$= P(j,k,m) * [P(j,k,m)^T * P(j,k,m)]^{-1} * P(j,k,m)^T \qquad (63)$$

Here, Q(I, k, m) depends on indices j, k and m.

Then Eq. (62) becomes,

$$P(j,k,m) * \begin{bmatrix} n_j \\ n_k \\ n_m \end{bmatrix} = Q(j,k,m) * U \qquad (64)$$

From Eq. (60), the LHS of Eq. (64) is U. Therefore, from Eqs. (64) and (60),

$$Q(j,k,m) * U = U \qquad (65)$$

Since U is known, the indices j, k and m are so chosen as to satisfy Eq.(65). Once j, k, m are determined, P(j, k, m) is obtained using Eq. (59). From P(j, k, m) the noise spike vector $[n_j\;\; n_k\;\; n_m]^T$ is calculated using Eq. (61). Then, the noise vector N_c is constructed. Then B_c, B_{1c} and B_{2c} are calculated as indicated by Eqs. (46), (12) and (13) respectively. Next, the conditions specified by Eq. (57) are verified for correctness of the j, k, m and N_c values. If Eq.(57) is not satisfied, the next set of j, k, m should be tried and so on. The algorithm is given below.

Algorithm 3.

```
for j = 1 to 6
  for k = j+1 to 7
    for m = k+1 to 8
      get P(j, k, m) as  P(j, k, m) = [Rj   Rk  Rm].
       get Q(j, k, m) as given by eq. (63)
     check whether this Q(j, k, m) satisfies Eq. (65) or not.
         If  Q(j, k, m) satisfies Eq. (65),
           use this j, k, m  and
            get [nj nk nm]^T by Eq. (60).
            Knowing j, k, m, and nj, nk and nm,
            construct the noise vector Nc.
            Calculate Bc as indicated byEq.(46).
        Get B1c and B2c from Bc as in Eqs. (12) and (13).
            Check whether Eq. (57) holds true .
            ( Check conditions, B1c=0 and abs(B2c) =1)
            If true,
               this Nc is and Bc are the correct answers.
               That is, B = Bc and N = Nc .  Exit.
            end if
         end if
         // go to next j, k, m iterations
      end for   // for  k
    end for    // for  j
  end for     // for m
```

A. *Example 3*

Let $N = [\,0\;\; -5\;\; 0\;\; 5\;\; 5\;\; 0\;\; 0\;\; 0]^T$

and $B = [\,0\;\; 0\;\; 0\; 1\; 0\;\; -1\;\; 1\;\; 1]$. Here, j =2, k = 4 and m =5.

From Eq. (8), $Y = S*B+N = [2\;\; -5\;\; -4\;\; 7\;\; 5\;\; -2\;\; 2\;\; 0]^T$.

From Eq. (24), $Z = S^T*Y = [5\;\; 5\;\; -5\;\; 23\;\; -5\;\; -13\;\; -7\;\; 13\,]^T$.

From Eqs.(26) and (27), U and V are given by,

$U = [5\;\; 5\;\; -5\;\; -5\,]^T$ and $V = [\,23\;\; -13\;\; -7\;\; 13\,]^T$.

In this example, 10 set of [j k m] values satisfy Eq. (54) before hitting the correct answer.

These set are, [1 2 7], [1 2 8], [1 3 5], [1 3 7], [1 4 7], [1 5 7], [1 6 7], [1 7 8], [2 3 6], [2 3 7] . Then the eleventh set [2 4 5] gives the correct answer. For this correct set, P(2, 4, 5) from Eq. (59) is,

$$P(2,4,5) = [R_2\; R_4\; R_5] = \begin{bmatrix} +1 & +1 & +1 \\ -1 & -1 & +1 \\ +1 & -1 & +1 \\ +1 & +1 & -1 \end{bmatrix}$$

From Eq. (63), Q(2, 4, 5) is,

$$Q(2,4,5) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & -0.5 \\ 0 & 0 & 1 & 0 \\ 0 & -0.5 & 0 & 0.5 \end{bmatrix}$$

VIII. CONCLUSIONS

The proposed technique with 8 data channels can detect and cancel up to 3 noise spikes. For 4 (or more) noise spikes, the rank of the corresponding P matrix (see Eq.(59)) is found to be 3 only. Hence our method cannot be applied to find the corresponding N. Our method can be extended for more number of data channels like K =16, 32 etc. When K =16, data channels 1, 2, 3, 5, 9 are held permanently at zero.

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