New Aggregation Operator for Triangular Fuzzy Numbers based on the Arithmetic Means of the L- and R- Apex Angles

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Abstract— **In this paper a new aggregation operator for a class of LR Fuzzy Numbers i.e., Triangular Fuzzy Numbers (TFNs) in which the L- and R- apex angles of the piecewise continuous linear membership function of the composite or resultant or aggregate TFN are the arithmetic means of the corresponding L- and R- apex angles of the individual TFNs is proposed. The L- and R- apex angles have been treated independently. Computation of the aggregate is demonstrated with a numerical example. Corresponding arithmetic and geometric aggregates have also been computed.**

Keywords— **LR Fuzzy Number, Triangular Fuzzy Number, Apex Angle, L-Apex Angle, R-Apex Angle, Aggregation Operator, Arithmetic and Geometric Mean**

I. INTRODUCTION

Many different types of fuzzy numbers are defined in the literature dealing with fuzzy logic and applications. In this paper only one class of fuzzy numbers i.e., Triangular Fuzzy Numbers (TFNs) which are a special class of LR fuzzy numbers are treated.

A. Triangular Fuzzy Numbers

TFNs are extensively used in fuzzy applications owing to their simplicity. TFNs are used in fuzzy applications where uncertainty exists on both sides of a value or parameter [1]. TFNs are characterized by an ordered triplet of real numbers $<l,m,u>$. Fig.1 depicts a TFN (l,m,u) with values v on x-axis and membership or grade μ along y-axis. The L- and R- apex angles are shown with arcs having one and two dashes respectively. Vertices of the triangle are at $(l, 0)$, $(m, 1)$ & $(u,0)$ moving clockwise. Line between $(l, 0)$ and $(m, 1)$ and between $(m, 1)$ and $(u,0)$ are the membership functions for the values in the intervals $[l, m]$ and $[m, u]$ respectively. Membership function of the TFN is the piecewise continuous linear. function represented by the lines L and R respectively. Intuitive meaning of such TFN is that the fuzzy number is approximately or around m [2]. The value of the fuzzy number varies in the range l and u. The possibility or membership grade of the number being a specified value v between l and u, $v \in [l, u]$ is represented by the ordinate of the projection of v on L or R accordingly as $l \leq v \leq m$ or $m \leq v \leq u$. $[l,m]$ is the range or interval which depicts or represents the possibilities of the number being a specific value less than m and $[m, u]$ is the interval which depicts the possibilities of the number being a

specific value greater than m. Possibility takes on the maximum value of 1 at m and reduces with increasing distance on either side (left / right) of m, becoming zero beyond l at the left and m at the right respectively.

Mathematically, a TFN is defined as follows. Let *l*, *m*, *u* $\in \mathbf{R}$, $l < m < u$. The fuzzy number *t*: $\mathbf{R} \rightarrow [0, 1]$ denoted by

$$t = \begin{cases} 0, & \text{if } x < l \\ \dfrac{x-l}{m-l}, & \text{if } l \leq x \leq m \\ \dfrac{u-x}{u-m}, & \text{if } m \leq x \leq u \\ 0, & \text{if } x > u \end{cases}$$

is called a triangular fuzzy number [1][2][3][4].

II. FUZZY AGGREGATION

Aggregation operations on fuzzy numbers are operations by which several fuzzy numbers are combined to produce a single fuzzy number [5]. An excellent account of Mathematical Aggregation Operators is given in [6]

A. Arithmetic Mean

The arithmetic mean aggregation operator [5][7] defined on n TFNs $<l_1,m_1,u_1>$, $<l_2,m_2,u_2>$, …, $<l_i,m_i,u_i>$, …, $<l_n,m_n,u_n>$ produces the result $<\bar{l}, \bar{m}, \bar{u}>$ where

$$\bar{l} = \frac{1}{n}\sum_1^n l_i \ , \ \bar{m} = \frac{1}{n}\sum_1^n m_i \ , \text{ and } \bar{u} = \frac{1}{n}\sum_1^n u_i$$

B. Geometric Mean

The geometric mean aggregation operator [5][8] defined on n TFNs $<l_1,m_1,u_1>$, $<l_2,m_2,u_2>$, …, $<l_i,m_i,u_i>$, …, $<l_n,m_n,u_n>$ produces the result $<\bar{l}, \bar{m}, \bar{u}>$ where

$$\bar{l} = \left(\prod_1^n l_i\right)^{\frac{1}{n}} , \ \bar{m} = \left(\prod_1^n m_i\right)^{\frac{1}{n}} , \text{ and } \bar{u} = \left(\prod_1^n u_i\right)^{\frac{1}{n}}$$

Other aggregation operators have also been defined in literature. For examples see [8][9][10][11].

C. Applications of Aggregation

The combination/aggregation/fusion of information from different sources is at the core of knowledge based systems.

Applications include decision making, subjective quality evaluation, information integration, multi-sensor data fusion, image processing, pattern recognition, computational intelligence etc. An application of aggregation operators in fuzzy multicriteria decision making is discussed in [7][8]. Another application in sensor data fusion is discussed in [10].

D. Organization of the Paper

In this paper a new aggregation operator for TFNs based on arithmetic means of apex angles is proposed. The L- and R- apex angles have been treated independently. The operator is described in the next section. In the subsequent sections, geometric interpretation and a numerical example is taken up. Results from the proposed operator are presented alongside those obtained from arithmetic and geometric mean aggregate operators.

III. PROPOSED AGGREGATION OPERATOR

Consider the TFN shown in Fig. 1. The most likely value of this TFN is m where the possibility $\mu = 1$. The apex angle of this TFN is $\mathcal{L}$lpu
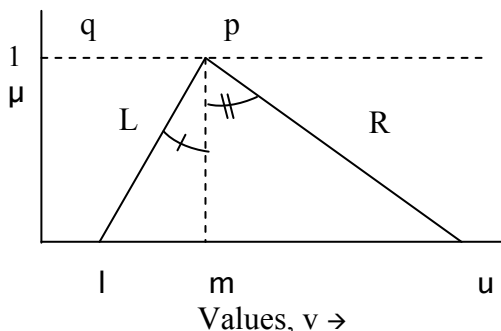

FIG.1: TRIANGULAR FUZZY NUMBER

However, $\mathcal{L}\, lpu = \mathcal{L}lpm + \mathcal{L}upm$

But, $\mathcal{L}lpm = \pi/2 - \mathcal{L}mlp$

$\mathcal{L}$ lpm is the contribution of the left-side line L to the apex angle referred to here as the L-apex angle.

Averaging over n TFNs we have

$$\frac{1}{n}\sum_{1}^{n}(\mathcal{L}\, lpm)_i = \frac{1}{n}\sum_{1}^{n}(\mathcal{L}\, (\pi/2 - mlp_i)$$

$$\frac{1}{n}\sum_{1}^{n}(\mathcal{L}\, lpm)_i = \pi/2 - \frac{1}{n}\sum_{1}^{n}(\mathcal{L}\, mlp)_i$$

The left side of the above equation represents the contributions of the left lines i.e., L's of all TFNs to the aggregate apex angle. It can be easily shown that

$$\tan\left(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}\, lpm)_i\right) = \frac{1}{\tan\left(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}\, mlp)_i\right)}$$

It can be shown that

$$\tan\left(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}\, mlp)_i\right) = \left(\tan\left(\frac{1}{n}\sum_{1}^{n}tan^{-1}((m_i - l_i))\right)\right)^{-1}$$

This represents the slope of the resultant aggregate left line $\bar{L}$. Similarly, it can be shown that the slope of the resultant fuzzy aggregate right line $\bar{R}$ is

$$-\left(tan\left(\frac{1}{n}\sum_{1}^{n}tan^{-1}((u_i - m_i))\right)\right)^{-1}$$

Under identical mathematical treatment, it can be shown that $\bar{m} = \frac{1}{n}\sum_{1}^{n}m_i$. Subsequently it is possible to show that

$$\bar{l} = \frac{1}{n}\sum_{1}^{n}m_i - tan\left(\frac{1}{n}\sum_{1}^{n}tan^{-1}((m_i - l_i))\right), \text{ and}$$

$$\bar{u} = \frac{1}{n}\sum_{1}^{n}m_i + tan\left(\frac{1}{n}\sum_{1}^{n}tan^{-1}((u_i - m_i))\right)$$

IV. NUMERICAL EXAMPLE

Consider the two triangular fuzzy numbers <1,1.5,3> and <5,6.5,9> aggregate TFN is computed as $\bar{m}$ =4; $\bar{l}$ =4– tan(0.5*(tan^{-1}(0.5)+tan^{-1}(1.5))=3.117; Similarly, $\bar{u}$ can be computed as 5.90.

Thus we have the aggregate as **<3.117,4,5.90>.** The arithmetic mean aggregate is **<3,4,6>** and the geometric mean aggregate is **<2.24,3.12,5.19>** respectively.

V. CONCLUSIONS

In this paper we have defined a new aggregate of triangular fuzzy numbers based on the arithmetic mean of the apex angle. The L- and R-apex angles have been treated independently. A numerical example has been worked out. The aggregate is the resultant piecewise continuous membership function line whose L- and R- apex angles are the arithmetic means of the L- and R- apex angles of the individual TFNs. The suitability of the aggregation operator proposed in this paper in different fuzzy logic applications involving fuzzy number aggregation remains to be explored.

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