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An Approximation to the Interval valued Fuzzy set

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Abstract: In this paper Interval Valued fuzzy set is obtained over the constant density function between the limits 0 to 1by using Series expansion and continued fraction expansion and is represented by the segment between the two membership functions.

Key words: Uncertainity,intervals.Points, Membership functions

1.INTRODUCTION

Interval valued fuzzy set is a membership function where we accept the uncertainity between the upper and lower bounds or we accept in terms of membership function drawn over the constant density function . In other words in terms of α cut

$A_\alpha = [\alpha_1, \alpha_2]$ where α_1 denote the value of the lower membership function and α_2 denote the value of the upper membership function. In this paper we have obtained the membership functions of the interval valued fuzzy sets by taking intervals and complementary functions at points. These sets are defined by functions of the form A:X $\to [0,1] \subset P[0,1]$ which is a fuzzy measure and fuzzy integral .Interval valued fuzzy sets by taking intervals have been studied by Klir and Yuan[1] and Incomplete Gamma function integral [2,3].Interval Valued Fuzzy Set can also be obtained for more than two values of the parameter where we obtained fuzzy sets .Works on Fuzzy Statistics have started within the last few years(see eg.Ganesh (2008), Pawlak(1985) Vasuki(1998) andYager(1982)) In this paper we have obtained the Interval valued fuzzy set by using series expansion and continued fraction expansion for two different values of the parameter.It is an approximation to the Interval valued fuzzy set. We put forward one such example through intervals and at points for the parameter p=.5 and p=1.

2..INCOMPLETE GAMMA FUNCTION INIEGRAL

For given values of x(>0) , p(>0) and the complete gamma function Γp the incomplete gamma is defined by

$$I(x,p) = \frac{1}{\Gamma p} \int_0^x e^{-t} t^{p-1} dt$$

The Series expansion is given by

I(x,p)=

$$\frac{e^{-x} x^p}{\Gamma p+1}\left[1+\sum_{r=1}^{\infty} \frac{x^r}{(p+1)(p+2)\ldots(p+r)}\right] \ldots(1)$$

For p≤ x ≤ 1 and also for x<p and for other cases the continued fraction expansion as shown below is used

$$\textbf{I(x,p)}=1-\frac{e^{-x}x^p}{\Gamma p}\left[\frac{1}{x+}\,\frac{1-p}{1+}\,\frac{1}{x+}\,\frac{2-p}{1+}\,\frac{2}{x+}\right] \quad \ldots\textbf{(2)}$$

3.FUZZY MEASURE AND FUZZY INTEGRAL

Let A(x) be a Interval Valued Fuzzy Set from [0,1] is called the core of the fuzzy set A(x) when its value is 1and is called support of A(x) i.e supp{A(x)}=A(x) for any crisp set A(x) when the set $\{x \in U / A(x) > 0\}$.Thus Interval Valued Fuzzy Set is a Fuzzy measure and Fuzzy integral over the range [0,1] which satisfies

(1) boundness : i.e g(Φ) =0 and g(X) =1

(2) monotonicity i.e A<B implies g(A) <g(B)

(3)continuity: $A_n \uparrow$ (or $\downarrow$)A implies that

$$Lt_{n\to\infty} g(A_n) = g(A)$$

Then g is called a fuzzy measure

Condition (3) can be discarded when the universal set is finite.

And Fuzzy integral which is defined by a fuzzy measure space (X,A,g) and let f:X→[0,1] be a measurable function on (X,A) and fuzzy integral of f over A(A∈ A)denoted by $\int_A fdg$ is defined as $\int_A fdg = \vee_{\alpha\in[0,1]}(\alpha \wedge g(f_\alpha \cap A))$ where f_α denote the α cut of f and convergence also occur at the Fuzzy measure and Fuzzy interval.It is called normal when its value is one.

4. APPROXIMATION TO THE INTERVAL VALUED FUZZY SET

The interval valued fuzzy set is obtained by taking intervals from [o,.5] by series expansion and from [.5 ,1] by using continued fraction expansion for the parameter p.

Taking x=.3 p=.5 r=1 in equation (1) we get

I(x,p)=0 , **I(x,p)**=.5 , 0≤x≤.3

which is continuous and strictly increasing

Next using the function

I(x,p)=

$$\frac{e^{-x} x^p}{\Gamma p+1}\left[1+\sum_{r=1}^{\infty} \frac{x^r}{(p+1)(p+2)\ldots(p+r)}\right] + x^2 \ldots(3)$$

Taking p=.5,r=1 in (3) we get

I(x,p)=.59 ,I(x,p)=.8 , .3≤x≤.4
which is continuous and strictly increasing
Again using the function
I(x,p)=

$$\frac{e^{-x}x^{p}}{\Gamma p+1}\left[1+\sum\frac{x^{r}}{(p+1)(p+2)...(p+r)}\right]+x^{2}$$

Taking p=.5,r=7 in (3) we get

I(x,p)= $\frac{e^{-.4}.4^{..5}}{\Gamma 3/2}\left[1+.360589527\right]+.16$

=.810734248

I(x,p)= $\frac{e^{-.5}.5^{.5}}{\Gamma 3/2}\left[1+.495526385\right]+.25$

=.973601595

I(x,p)=.810734248,I(x,p)= .973601595, .4≤x≤.5
which is continuous and increasing

For the interval[.5,1] we use the continued fraction expansion at points

Taking x=.75 p=.5 in (2) we get
I(x,p)=1-

$$\frac{e^{-.75}.75^{.5}}{\Gamma 3/2}\left[\frac{5x^{2}+x^{2}-px+2}{6x^{2}+x^{3}-2px^{2}+6x-4xp}\right]$$

=.978559813

Next taking x=.5 and p=.5 in (2) we get

I(x,p)=1- $\frac{e^{-.5}.5^{.5}}{\Gamma 3/2}\left[\frac{5x^{2}+x^{2}-px+2}{6x^{2}+x^{3}-2px^{2}+6x-4xp}\right]$

=.533400765+.5=1

Since we are using the continued fraction for half of the interval we are adding with .5

Taking x=1,p=.5 in equation (2) we get
I(x,p)=1-

$$\frac{e^{-x}x^{p}}{\Gamma p+1}\left[\frac{5x^{2}+x^{2}-px+2}{6x^{2+}x^{3}-2px^{2}+6x-4xp}+\frac{x+1}{x(x+2-p)}\right]$$

…(4)

=.00032014≈0

Next we have obtained the fuzzy valued membership function for the parameter p=1

For the interval[o,.5]

Taking x=.3,p=1,r=2 in equation (1) we get

I(x,p)=0, I(x,p)=.3 ,0≤x≤.3

Taking p=1,r=1 in equation (3) we get

I(x,p)=.345582286 , I(x,p)=.481753622 ,.3≤x≤.4

Next using the equation

I(x,p)

$$=\frac{e^{-x}x^{p}}{\Gamma p+1}\left[1+\sum\frac{x^{r}}{(p+1)(p+2)...(p+r)}\right]+x^{2}+.01k \quad \ldots(5)$$

Taking r=1,p=1 in equation (5) we get
I(x,p)=.48753622, I(x,p)=1 ,.4≤x≤.5
For p=1 ,x=.5 for the interval [.5,1] in equation (2) we get

I(x,p)=.995512527≈1

For p=1,x=.75 in equation (2) we get

I(x,p)=.527633447

Next using the equation
I(x,p)=1-

$$\frac{e^{-x}x^{p}}{\Gamma p+1}\left[\frac{5x^{2}+x^{2}-px+2}{6x^{2+}x^{3}-2px^{2}+6x-4xp}+\frac{x+1}{x(x+2-p)}\right]$$

+.7x^2 …(6)

For p=1 ,x=1 in equation (6) we get

= .006725509≈ 0

Finally if you sum up all the intervals for the parameter p=.5 we get

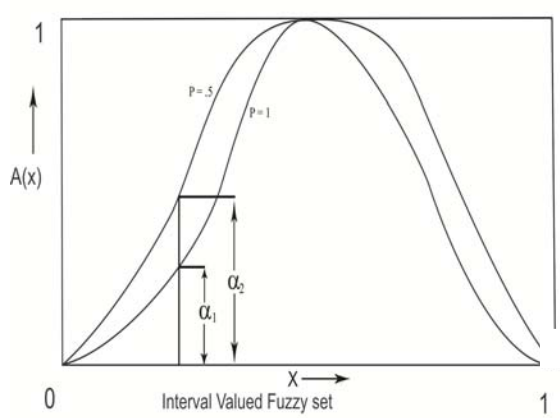
$$I(X,P)=\begin{cases} 0 \quad ,5 \quad , \quad 0\le X\le .3 \\ .59 \quad ,.8 \quad ,.3\le X\le .4 \\ .810734248 \quad ,1 \quad ,.4\le X\le .5 \\ 1 \quad , X=.5 \\ .978559813 \quad , \quad X=.75 \\ .00032014 \quad , \quad X=1 \end{cases}$$

Similarly if we add up all the intervals for the parameter p=1 we get

I(X,P)=

$$\begin{cases} 0 \quad , \quad .3 \quad , \quad 0\le X\le .3 \\ .345582286 \quad ,.481753622 \quad .3\le X\le .4 \\ .48753622 \quad ,1 \quad .4\le X\le .5 \\ .995512527 \quad , \quad X=.5 \\ .527633447 \quad , \quad X=.75 \\ .006725569 \quad , \quad X=1 \end{cases}$$

which is shown below with the help of diagram.



5.CONCLUSION:

From the interval valued fuzzy set we can obtained the Fuzzy Measure , monotoncity, continuity and each probability measure P is a Fuzzy measure and we also obtained the fuzzy integral and convergence also occur at the Fuzzy measure and Fuzzy integral. From the

Interval Valued Fuzzy Set we can obtain Fuzzy Set by allowing their Intervals to be fuzzy .

$$F(Y) = .1771109Y^4 + .2989351Y^3 + 1.0811693Y^2 - .8855745Y + .1266906 \quad ,13/16 \le Y \le 7/8$$
$$F(13/16) = .3782421 \qquad ,F(7/8) = .5$$
$$i.e Nec(.3762421 \cap .5) = \min[Nec(.376242), Nec(.5)] = .376242$$
$$and Pos(.3762421 \cup .5) = \max[Pos(.376242)]$$

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