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A Fuzzy-AHP Approach to Solve Multi Criteria Budget Allocation Problem

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Abstract— **Decision-making problems require systematic approach to evaluate alternatives using both quantitative and non-quantitative factors. Standard methods to solve problems lack considerations of non-quantitative factors, in which numeric value is difficult to assign. Different techniques like Fuzzy set theory, Analytic Hierarchy Process (AHP) and Multi Criteria Decision Making (MCDM) are presently being used in decision-making process. These techniques take multiple factors with vague values and /or concrete values. This paper provides solution to a decision-making problem of budget allocation problem, to allocate funds to deserving and competing organizations by using integrated Fuzzy, AHP and MCDM techniques. In budget allocation problem, fund seekers submit their proposals to avail funds to fund allocator to execute their projects. Fund allocators allocate funds to fund seekers after evaluating their proposals. During evaluation, 12 decision-making factors are considered and these are given weights. Weights are calculated using Fuzzy set theory and AHP. Fuzzy set takes subjective values like preferred, strongly preferred etc. and AHP technique evaluates relative importance of factors by forming pairwise comparison matrix. Experts in this domain were consulted to give their preferences through questionnaire. The technique of evaluating proposals helped in ranking after assigning weights to decision-making factors.**

Keywords - AHP, MCDM, Fuzzy, Budget Allocation.

I. INTRODUCTION

Problem solving is set of activities designed to analyse problem systematically and provide valuable solution. Decision-making is mechanism for making choices during every step of problem solving. Standard problem solving models ensure that decisions made are logical and rational. During decision-making, quantitative and non-quantitative decision making factors are considered and evaluated. Quantitative factors provide a numerical value for making decision. Such factors are insufficient in decision-making process, hence non-quantitative factors like SWOT (Strength, Weakness, Opportunity, Threat) and PEST (Political, Economical, Social, Technology) analysis are also considered. Such non-quantitative factors are not easy to evaluate in terms of numeric value. Decision-making models to solve problems are classified into three categories: Decision Theory Approach, Economic Analysis and Operational Research Approach [1]. Decision Theory models rely on subjective or qualitative input variables, Economic Analysis is based on probability in terms of investment required and expected revenue. It makes use of methods like NPV (Net Present Value) and Discounted Cash Flow and assumes that profit is the only objective. NPV and other discounted cash flow methods are inappropriate in research and development project selection as they favor short-term projects not long-term projects where market is uncertain [2]. Problem arises when non-economic benefits are considered. The Operational Research uses mathematical programming techniques to optimize selection of alternatives, provided constraints and other resources are available.

During ongoing research, a solution to decision-making problem to allocate budget to deserving and competing fund seeker is provided. In India, lots of funds are allocated to organizations executing their projects like UGC (University Grant Commission) allocates funds in education sector to provide quality education, DIT (Department of Information Technology) provide funds to promote Research and Development activities in scientific organizations etc. The success of these projects depends on many unforeseen situation. This paper focuses on decision-making technique to allocate budget after considering both quantitative and non-quantitative factors and mechanism to convert non-quantitative factors into quantitative. The structure of the paper is as follow:

Section II briefs the background and related work in this area. Section III discusses the methodology adopted to solve problem. Section IV details criteria used in evaluating proposals. Section V shows the result of data compilation. Section VI describes the procedure to determine weights and rank followed by conclusion in section VII.

II. BACKGROUND AND RELATED WORK

Budget allocation problem occurs when limited funds are to be allocated to most deserving and competent fund seekers. Lots of funds are allocated in various areas like education, research & development and social oriented schemes. Funds seekers submit project proposals and these proposals are filtered according to criteria set by fund allocator. The proposals after matching criteria are then evaluated technically and financially. Both quantitative and non-quantitative decision-making factors are considered to rank the projects. Weightage is given to each decision-making criterion. After ranking, budget is allocated according to availability and rank [3].

One approach to solve such decision-making problems is use of MCDM, which involves the identification of multiple criteria involved and assigning weights to them. Qualitative multi criteria is facilitated by peer group evaluation and rating. The alternatives are ranked on composite score. MCDM in combination with fuzzy set theory has also been used by many researchers in past to make decision. Fuzzy set theory is used when data is vague, incomplete or uncertain. A similar solution is provided in selection procedure of public sector projects in Taiwan [4]. Fuzzy logic allows intermediate values to be defined rather than only true/false or yes/no. Values like good, very good, poor and very poor can be formulated mathematically and processed by computers. In this way, a more human like behavior can be simulated in computer. Fuzzy-MCDM has been in use to make decision of selection of infrastructure projects in three stages [5].

Another approach to solve decision-making problem using multi-criteria is AHP introducted by Saaty. It has now become most widely used decision-making method. It involves six steps: Defining unstructured problem, planning hierarchical structure, applying pairwise comparisons, use of Eigen values, checking consistency and aggregating relative weights [6]. AHP is also useful when factor is further divided into sub factors. The hierarchy of criteria depicts top-down managerial concept and bottom-up evaluation of criteria. Lowest level factors are calculated first and then are aggregated at higher level. While fuzzy logic is suitable for evaluating subjective factors, AHP is suitable for comparing alternatives rather than merely ranking. Integration of fuzzy logic with AHP gives advantage of making subjective factors more quantifiable [7, 8]. Based on background and related work, it is observed that integrated approach of Fuzzy, AHP with MCDM is useful in solving in the budget allocation problem.

III. METHODOLOGY ADOPTED

To solve the budget allocation problem, fund allocation procedures of funding agencies like DIT and DST (Department of Science and Technology) etc. were studied and analyzed. Thereafter, a methodology was adopted to solve the problem, steps of which are described below:-

A. Identification of Decision-Making Factors

After reviewing the proposal submission forms of funding agencies, twelve decision-making factors are identified. A three level hierarchy is designed for these factors.

B. Data Collection and Compilation

After identification of factors, next step was to prepare a well-structured questionnaire. This was used to get preferences of factors from the domain experts. The filled questionnaire is used to prioritize the factors and to calculate weightage of each. While preparing questionnaire, a combination of Fuzzy logic and MCDM is considered. The questionnaire was uploaded on web site. Experts were contacted and requested to fill the questionnaire. Data is collected from domain experts in person also. Collected data is compiled to get result.

C. Determining Weights and Ranking Proposals

The collected data was analyzed using AHP technique. A fuzzy-AHP with MCDM is used to find the weightage of each factor.

IV. DECISION-MAKING FACTORS HIERARCHY

During research, twelve decision-making factors were identified. These 12 factors are classified in five broad categories. The detail of such factors is given below:-

A. Solution Delivery and Contribution

This factor is concerned with the relevance of the project and its contribution to society.

- *Core Area:* Core area is concerned with the prime objective of the organization.
- *Human Resource Development:* This factor is related with the number of persons trained or will get employment as a result of this project.
- *National Development and Impact on Society:* Contribution to nation as a whole and impact on society or section of society on successful implementation of project is considered.

B. Technical

This factor addresses the technical issues related with implementation of the project. Technical covers technology available and expected success rate.

- *Technology Availability:* This factor is concerned with the availability of technology to be used in the project
- *Success Probability:* Success probability of completion of the project is key factor in decision-making.

C. Financial

This factor addresses the financial aspects involved in the project like cost and economical benefits.

- *Cost Involved:* Cost involved in the project is considered.
- *Economical Benefit:* Expected benefits in monetary form as a result of the execution of the project are evaluated.

D. Capacity and Expertise

This factor is concerned with capacity and capability of the organization in handling the projects/ schemes.

- *Infrastructure:* Infrastructure (Hardware, Software, space, labs, etc) already available with organization/individual are considered.
- *Management Capability:* Management capability of the organization to execute the project is judged.
- *Staff Experience and Qualification:* This considers skills and qualification of the staff involved in the project.

E. Risk Management

Risk Management covers the risk factors involved like completion and implementation risks.

- *Project Completion:* Project Completion Risk identifies any unforeseen circumstances leading to delay in delivery.
- *Implementation Risk*: This is concerned with risk involved in implementation.

A three level hierarchy of factors is shown in figure 1. Level 1 shows the ultimate goal to achieve and that is, to evaluate the proposals. This is further divided into five categories and each category has further 2 or 3 sub factors. So far, evaluation of proposal is concerned, it is in reverse order. First level 3 factors are evaluated and then level 2 factors are evaluated as accumulation of level 3 factors. Similarly, level 1 is evaluated as aggregation of level 2 factors.

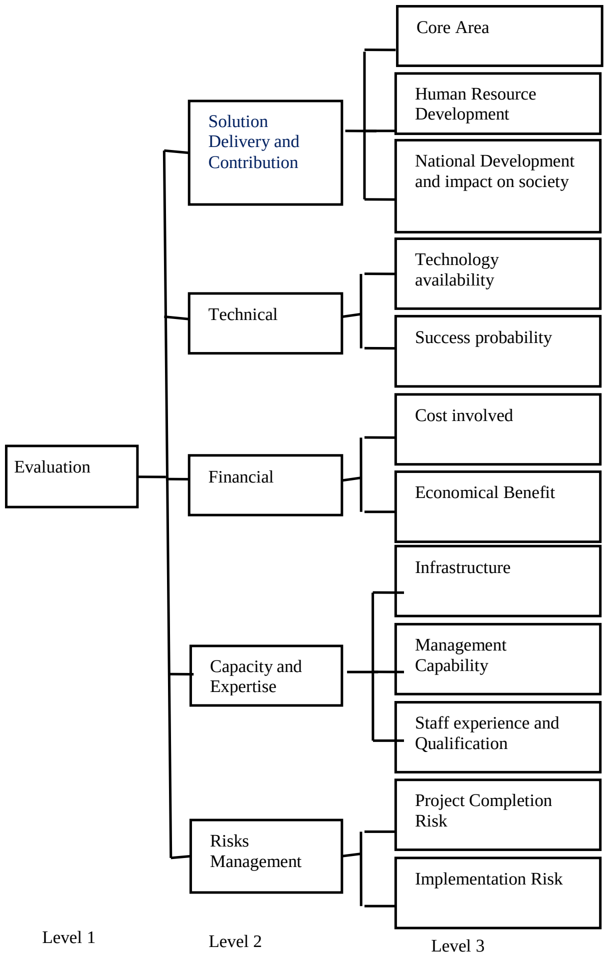

Fig. 1: Decision Making Factors' Hierarchy

V. DATA COMPILATION

Based on above decision-making factors, a questionnaire was prepared and domain experts were consulted for their views on preferences of factors. Experts were requested to give one preference out of five options: Least Preferred, Moderately Preferred, Strongly Preferred, Very Strongly Preferred and Extremely Preferred, having importance on numerical scale as 1,3,5,7 and 9 respectively. Table I shows the result of data compilation. The Geometric mean of the data collected is calculated against each factor. To get large values, the Geometric mean is multiplied by 1000 as shown in column (D).

TABLE I
COMPILED DATA

Factor Id (A)	**Factor Description (B)**	**Geometric Mean (C)**	**(C)* 1000 (D)**
c11	Core Area	8.511000	8511
c12	Human Resource Development	6.079000	6079
c13	National Development and Impact on Society	6.617000	6617
c21	Technology Available	6.435000	6435
c22	Success Probability	6.198000	6198
c31	Cost Involved	4.022000	4022
c32	Economic Benefit	5.381000	5381
c41	Infrastructure	5.856000	5856
c42	Management Capability	6.252000	6252
c43	Staff Experience and Qualification	7.130000	7130
c51	Project Completion Risk	5.486000	5486
c52	Implementation Risk	4.678000	4678

VI. DETERMINING WEIGHTS AND RANKING PROPOSALS

After collecting the experts' views and compiling collected data, next step was to determine the weights of each factor. Weights are determined by applying AHP technique. First, a pairwise comparison matrix is designed. A pairwise comparison matrix, square matrix, compares the importance of one alternative over other [9]. The pairwise comparison matrix is shown in Table II below:-

TABLE II
PAIRWISE COMPARISON MATRIX (M)

	c11	c12	c13	c21	c22	c31	c32	c41	c42	c43	c51	c52
c11	1	1.400066	1.286232	1.322611	1.373185	2.116111	1.581676	1.453381	1.361324	1.193689	1.551404	1.819367
c12	0.714252	1	0.918694	0.944678	0.9808	1.511437	1.129716	1.038081	0.972329	0.852595	1.108093	1.299487
c13	0.777464	1.088501	1	1.028283	1.067602	1.645201	1.229697	1.129952	1.058381	0.92805	1.206161	1.414493
c21	0.75608	1.058562	0.972495	1	1.038238	1.59995	1.195874	1.098873	1.029271	0.902525	1.172986	1.375588
c22	0.728234	1.019576	0.936678	0.96317	1	1.541024	1.151831	1.058402	0.991363	0.869285	1.129785	1.324925
c31	0.472565	0.661622	0.607828	0.625019	0.648919	1	0.747445	0.686817	0.643314	0.564095	0.733139	0.859769
c32	0.632241	0.885178	0.813208	0.836208	0.868183	1.337892	1	0.918887	0.860685	0.754698	0.98086	1.150278
c41	0.688051	0.963316	0.884993	0.910023	0.944821	1.455992	1.088274	1	0.93666	0.821318	1.067444	1.251817
c42	0.734579	1.028459	0.944839	0.971562	1.008712	1.554451	1.161866	1.067623	1	0.876858	1.139628	1.336469
c43	0.837739	1.17289	1.077528	1.108003	1.150371	1.77275	1.325033	1.217555	1.140435	1	1.299672	1.524156
c51	0.644578	0.902451	0.829077	0.852525	0.885124	1.363998	1.019513	0.936817	0.877479	0.769425	1	1.172723
c52	0.549642	0.769534	0.706967	0.726962	0.75476	1.163103	0.869355	0.798839	0.748241	0.656101	0.852716	1

To explain, above matrix, let us take the case of factor C11. Factor C11 is preferred over factor C12 with value 1.400066. Factor C12 has priority over C11 with numeric value as 0.714252. It is on the assumption that when factor i has some value assigned to it compared with factor j, then j has reciprocal value when compared with i. Diagonal elements have value one. AHP determines priorities of each factor or importance or weight by analyzing matrix using mathematical theory of Eigen values and Eigen Vectors. Eigen Vector is based on maximum Eigen value as weight. The whole calculation of AHP is done using MatLab, a tool for numerical computation and visualization [10]. Table III shows Eigen vectors E and weights/priority W.

The last stage in AHP is to calculate Consistency Ratio (CR) to measure consistency of data [11]. If CR is higher than 0.1, the judgments are untrustworthy and exercise is valueless. CR is calculated as CI/RI.

CI, Consistency Index is defined as:

$(\lambda_{max} - n)/(n-1)$, n = number of factors.

RI, Random Consistency Index, is derived by Saaty and for n = 12 its value is derived as 1.48 [12, 13].

Procedure to calculate λ_{max}, is as follow:-

Multiply pairwise matrix by Eigen Vector to get new vector. Then Eigen Vector divides this new vector and mean of this vector is λ_{max} as shown in Lambda Vector. Value of λ_{max} is measured as 12.0001 and CI comes out to be 0.0000091. Dividing CI by 1.48, CR comes out to be 0.000000, which is much less than 0.1 and hence the judgment is worthy and can be applied in budget allocation problem.

The weight W_i is obtained by normalizing E_i, i.e. transforming in such a way that resultant sum is one (unity). These values are used to prioritize the projects for funding. Graphically, share of each factor in percentage at level 3 in shown in figure 2

TABLE III

PRIORITY VECTOR

Value of i	Factor	Eigen Vector (E_i)	Priority /Weight (W_i)	Lambda Vector	(E)/ (D)
(A)	(B)	(C)	(D)	(E)	(F)
1.	c11	0.3993	0.117158617	1.405915	12.0001
2.	c12	0.2852	0.083680535	1.004178	12.00013
3.	c13	0.3104	0.091074467	1.093049	12.0017
4.	c21	0.3019	0.088580482	1.062985	12.00021
5.	c22	0.2908	0.085323631	1.023835	11.99943
6.	c31	0.1887	0.055366469	0.664386	11.99979
7.	c32	0.2525	0.074086028	0.888876	11.9979
8.	c41	0.2747	0.08059973	0.967341	12.00179
9.	c42	0.2933	0.086057156	1.032755	12.0008
10.	c43	0.3345	0.098145649	1.17779	12.00043
11.	c51	0.2574	0.075523737	0.906221	11.99916
12.	c52	0.2195	0.064403497	0.772749	11.99856

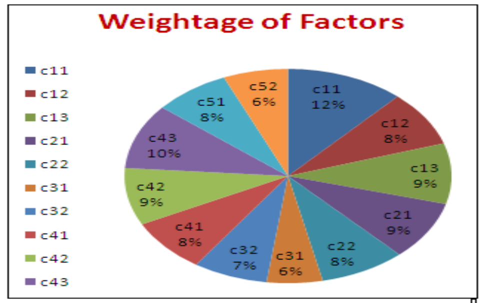

Fig. 2: Share of Level 3 factors in value 1

Share of level 2 factors is calculated by aggregating level 3 factors. Figure 3 shows share of such level 2 factors.

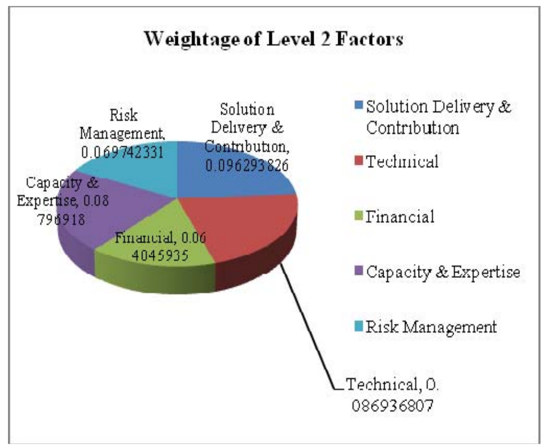

Fig. 3 Share of level 2 factors in value 1

The weights calculated above are used in making decision to allocate budget. The proposals are evaluated against factors mentioned above and weightage is given as per values computed. The proposals are then ranked and budget is allocated according to rank and weightage. A procedure to evaluate value of each factor from proposals submitted has been implemented in web based multi agent system for resource allocation and monitoring [14]. Here, three agents are designed; Fund Seeker Agent, Coordinator Agent and Fund Allocator & Monitor Agent. Coordinator Agent interacts with users of system through GUI, while Fund Seeker Agent receives proposal and validates it and Fund Allocator & Monitor Agent filters the proposals, evaluate these filtered proposals w.r.t factors mentioned above, assigns weights and then finally allocates funds [15].

Table IV shows the evaluation of three projects as a result of implementation of ranking process in multi agent resource allocation in web environment. Column 'C' shows the evaluated numeric values as a part of evaluation process implemented in allocation procedure. Column 'D' of table shows the weightage as per experts' view and as calculated above. Column 'E' is computed weight as result of multiplication of values of columns 'C' and 'D'. To explain, consider Project Id 48, its evaluated numeric value is 0.333 against criteria C12 and its weightage is only 11.7 percent, hence it contributes to .0390 share in unity during ranking. Graphically, it has been shown in figure 4.

After evaluating level 3 factors, next step is to evaluate level 2 factors. This is done by accumulating lower level factors to high level factors using Geometric Mean e.g. value of c1 factor is calcutated by applying Geometric Mean on values of c11, c12 and c13 factors. Similarly remaining factors are evaluated. The calculated values are shown in table V. Similarly Level 1 factors are calculated and projects are finally ranked. Project Id 48 gets rank 1, 61 gets rank 2 and 81 gets rank 3. The same has been shown graphically in figure 5.

TABLE IV

EVALUATION OF PROJECTS

Project ID	Criteria	Evaluated Numeric	Weightage	Computed Weight
(A)	(B)	(C)	(D)	(E)
48	c11	0.333	0.117	0.0390
48	c12	0.333	0.084	0.0279
48	c13	0.333	0.091	0.0303
48	c21	0.333	0.089	0.0295
48	c22	0.263	0.085	0.0224
48	c31	0.337	0.055	0.0187
48	c32	0.333	0.074	0.0247
48	c41	0.422	0.081	0.0340
48	c42	0.333	0.086	0.0287
48	c43	0.686	0.098	0.0673
48	c51	0.6	0.076	0.0453
48	c52	0.456	0.064	0.0294
61	c11	0.333	0.117	0.0390
61	c12	0.333	0.084	0.0279
61	c13	0.333	0.091	0.0303
61	c21	0.333	0.089	0.0295
61	c22	0.316	0.085	0.0270
61	c31	0.379	0.055	0.0210
61	c32	0.333	0.074	0.0247
61	c41	0.213	0.081	0.0172
61	c42	0.333	0.086	0.0287
61	c43	0.26	0.098	0.0255
61	c51	0.405	0.076	0.0306
61	c52	0.323	0.064	0.0208
81	c11	0.333	0.117	0.0390
81	c12	0.333	0.084	0.0279
81	c13	0.333	0.091	0.0303
81	c21	0.333	0.089	0.0295
81	c22	0.421	0.085	0.0359
81	c31	0.284	0.055	0.0157
81	c32	0.333	0.074	0.0247
81	c41	0.365	0.081	0.0294
81	c42	0.333	0.086	0.0287
81	c43	0.054	0.098	0.0053
81	c51	0	0.076	0.0000
81	c52	0.221	0.064	0.0142

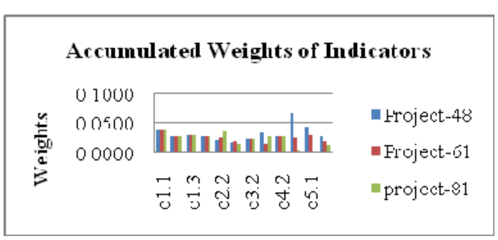

Fig. 4: Graphical representation of values of Factors

TABLE V

LEVEL-1 FACTORS

Criteria	Project-48	Project-61	project-81
c1	0.032066	0.032066	0.032066
c2	0.025728	0.028201	0.032551
c3	0.021455	0.022753	0.019696
c4	0.040336	0.023241	0.016471
c5	0.03648	0.025225	0.001193
GM	0.030419	0.026079	0.013221
Weights	0.436	0.374	0.19
Rank	1	2	3

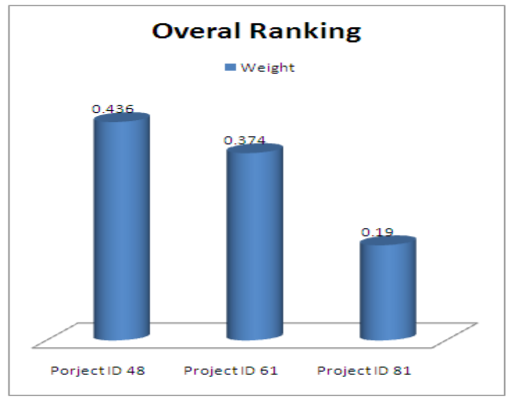

Fig. 5: Overall ranking of projects

VII. CONCLUSION

Decision to rank project proposals for budget allocation is complex since it involves multiple criteria. Some criteria have direct numeric value associated with it while others do not have. In such situation, fuzzy set theory with MCDM helps in evaluating subjective criteria. AHP helps in comparing projects and assigning weights or importance to them. Combination of these three techniques helped in solving decision-making problem of budget allocation. The proposals are ranked and given funds according to rank and availability.

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